# Floor-Kelly With Money In and Money Out

## Contributions, Withdrawals and Partner Assets for a Floor-Anchored Kelly Rule

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*Research on simulated worlds and walk-forward history. Not financial advice; no result here is a guarantee.*

*v1.4 corrects the simulation. Its 12-month blocks of shocks under-sampled the first and last 12 months of the sample, both of which had below-average shocks; that set simulated prices about 10% higher against the trend than the fitted model. Every simulated number was recomputed with circular blocks; about three in four withdrawal rates fall by 0.5 to 2 points, a quarter are unchanged and a few rise by up to 1 point; no registered verdict changes (Section 12). The historical results are unchanged.*

*Builds on Paper 19 (Kelly Sizing With Floor-Anchored Growth) and Paper 20 (Does a Frozen Power-Law Floor Hold?). Plan: pre-registration `research/F009-preregistration.md` (commit 6ad98e5) and research proposal `research/F009-proposal.md` v1.3, both written before the simulations ran. The proposal was accepted after independent review (9.0/10). What was known when is set out in Section 12. Drafted as F009 and reviewed under that name. New claims: C8.1 to C8.8 (Research Claims Registry, Domain 8, added in v2.8). Two later Observatory studies build on it: F010 (sell ladders, forthcoming as Paper 22) and F011 (spending the floor's growth, forthcoming as Paper 23).*

## Abstract

Paper 19 tested a Kelly rule for Bitcoin whose growth input is the growth from today's price to the power-law floor four years ahead. It was tested on a lump sum. This report tests the rule with money going in (monthly contributions, dollar-cost averaging) and money coming out (monthly withdrawals indexed to inflation). It also compares the floor input with three alternatives: the historical average return ("normal" Kelly), the trend's growth rate, and the trend itself as the target. Each runs at full, half and quarter Kelly, with gold, cash or US stocks next to Bitcoin.

The evidence comes from walk-forward history (2012 to 2026) and from 30-year simulated futures (16 cases, 2,000 paths each) in which price can stay below the floor. History has about three independent four-year windows and three cycle tops, so it describes rather than tests.

**History.**

- **Saving:** full floor-Kelly with DCA ended with a median 1.10× the Bitcoin of DCA into Bitcoin over 48-month windows (ahead in 78%), and 1.27× over 96 months (99%). It also had the highest certainty-equivalent wealth at every risk aversion tested.
- **Retiring:** from the 2013, 2017 and 2021 tops, half floor-Kelly survived withdrawal rates as high as 19% to 21% a year over the 5 to 13 years to October 2026 (not 30-year rates), against 10% to 13% for Bitcoin only. At 8% its real wealth fell to 44% to 56% of the start, against 11% to 19%.
- **The floor as a signal:** every 12-month return from below 1.25× the floor was positive (36 of 36).

**Simulation from today, if the power law continues.**

- **Saving:** with the floor refit inside each path, full floor-Kelly ended 20 years of DCA with a median 0.90× the Bitcoin of DCA into Bitcoin. Outcomes were widely spread (10th to 90th percentile 0.18× to 3.57×).
- **Why:** as price recovers toward the trend, the rule cuts its Bitcoin. Simulated crashes also go deeper than history's, and each one pulls the refit floor down, from 0.45× the trend today to about 0.24× after 25 years. Measured from that lower floor, the rule's growth input is smaller, and it holds less Bitcoin.
- **Retiring today:** in BASE, half floor-Kelly's 30-year safe withdrawal rate (SWR95: the highest starting rate that 95% of paths survive) was 10.5%, against 19.5% for Bitcoin only. These rates are high because the model assumes the power law holds (a median 480× rise in 30 years). With trend growth halved, Bitcoin only's was 9.5%; in a random walk, Bitcoin only had no safe rate.
- **Retiring at a peak (2.5× trend):** half floor-Kelly beat Bitcoin only by 2.0 points with cycles intact and 3.5 points with trend growth halved, but by only 0.5 points with cycles fading.
- **Trend-anchored Kelly** led most simulated cases. Its starting target equals the model's own four-year forecast (0.898× trend), so much of that lead is built in, although it also led where the trend is wrong. In history it trailed floor-Kelly at full Kelly.

**Where history and simulation agree:**

- savers did better with full floor-Kelly than with half or quarter floor-Kelly;
- a retiree starting at a top did better with half floor-Kelly than with Bitcoin only, mainly because it held less Bitcoin. This rests on an agreement test adapted after the results, and in simulation a fixed 50/50 Bitcoin and gold mix did as well or better. Stocks did better than gold as the partner, at 2012 to 2026 stock returns.

**Other findings.**

- **"Normal" Kelly** was close to Bitcoin only in history.
- **Reflecting the price at a floor of 0.432× trend**, the floor Papers 15 to 17 used, raises Bitcoin only's 30-year SWR95 from 19.5% to 30.0% from today, and from 5.0% to 6.5% from a peak (about 30% more). A later study (F011) found Paper 16's spending levels fail without it.

**Keywords:** Bitcoin, Kelly criterion, dollar-cost averaging, safe withdrawal rate, sequence risk, power law, gold, stocks, walk-forward, Monte Carlo.

## Terms

- **Trend:** the published power-law fit of Bitcoin's price against time ($\log_{10}P = -16.493 + 5.688 \log_{10}$ days since 2009-01-03). Price ÷ trend is the **multiple** (0.49 at the end of October 2026).
- **Published floor:** 0.42× the trend. **Walk-forward 1% floor:** a line fitted at each date, on daily closes up to that date, so that about 1% of them lie below it (a 1% quantile regression); floor-Kelly uses this one. In the simulation it is **refit** every 12 months on past prices plus each path's own prices, or **frozen** at its October 2026 fit.
- **Floor-Kelly (FK):** Kelly sizing with Bitcoin's growth input set to the growth from today's price to the walk-forward floor four years ahead. **Trend-anchored Kelly (TA):** the same rule aimed at

the walk-forward trend. **Historical Kelly (HK):** the textbook rule fed Bitcoin's average past return. **Trend Kelly (TK):** fed the trend's growth rate. Full, half and quarter are the Kelly fraction.

- **DCA:** a fixed real amount into Bitcoin each month. **Stack ratio:** the Bitcoin the portfolio is worth at the end (its value divided by the Bitcoin price), against the Bitcoin that DCA into Bitcoin holds at the end.
- **SWR95:** the highest first-year withdrawal, as a share of the starting portfolio and then raised with inflation, that at least 95% of simulated paths survive for 30 years.
- **Certainty-equivalent (CE) wealth:** the sure amount an investor with risk aversion $\gamma$ would accept instead of the uncertain outcome; a higher $\gamma$ puts more weight on bad outcomes.
- **Worlds:** BASE (cycles as fitted), DAMPED (shocks halved), FADING (shocks fade to 10% by year 30), NO_REVERSION (a random walk around the trend), BROKEN (half the trend's growth). Starts: TODAY (0.49× trend), TREND, PEAK (2.5× trend).
- **The owner:** the Observatory's owner, who set this study's questions. **P1 to P5:** the pre-registered tests (Section 9). **H1 to H3:** the registered historical tests (saving, retiring, the buying signal). **S1 to S8:** the secondary analyses added in the research proposal (its Section 7).

## Key findings

1. **Saving, history: full floor-Kelly adds Bitcoin on top of DCA.** It ended with a median 1.10× the Bitcoin of DCA into Bitcoin over 48 months and 1.27× over 96 months. Against its own average mix it gained 1.23× and 1.76×, so the gain is timing, not allocation (Table 2, Figure 2).
2. **Saving, simulation: with the floor refit inside each path, that edge does not continue.** Full floor-Kelly ended at a median 0.90× in BASE and below 1 in four of five worlds. Simulated crashes go deeper below the floor than history's: 16% of months below the published floor, against 2% in 2012 to 2026. Both the refit and the frozen version start all in and cut Bitcoin as price recovers from 0.49× toward the trend. Each crash then lowers the refit 1% line, and that explains the gap between them: 44% against 58% Bitcoin after five years, 19% against 49% after twenty-five (Figure 6). The floor's slower growth as Bitcoin ages adds to this (Figure 7).
3. **Retiring at a top: half floor-Kelly protected the retiree, mainly by holding less Bitcoin.** In history it survived 19% to 21% a year from each past top over the 5 to 13 years to October 2026, against 10% to 13% (Table 5). In simulation from 2.5× trend its SWR95 was 2.0 and 3.5 points higher than Bitcoin only's in two of three worlds (Table 7). History and simulation agree here, under an agreement test we adapted after the results were known. In simulation, a constant mix with the same average weights came within 0.5 point of it in all three worlds, and a fixed 50/50 Bitcoin and gold mix did as well or better (Section 5.2).
4. **Retiring today, near the floor: Bitcoin only beats floor-Kelly.** Its SWR95 was 19.5%, against 10.5% for half floor-Kelly (90% interval of the gap -9.5 to -9.0 points).
5. **"Normal" Kelly is close to Bitcoin only.** On Bitcoin's history since 2010, the plug-in formula held 100% Bitcoin at full Kelly and 90% on average at half Kelly. It added no Bitcoin.
6. **Floor or trend as the target: the trend led in simulation, the floor in history.** The same rule aimed at the trend instead of the floor:
    - led most simulated cases (20-year DCA 4.40× in BASE; SWR95 20.0% today and 10.0% from a peak at half Kelly);
    - trailed floor-Kelly in history at full Kelly (1.04× against 1.10×).

Its starting target, 0.898× trend four years out, equals the model's own expected path. In BASE, DAMPED and FADING it is therefore Kelly with the model's own forecast, and its lead there is built in. It also led in BROKEN, where the trend is wrong, probably because its refit trend adapts.

7. **For floor-Kelly, full beats half and quarter for savers, in history and in simulation**, at every risk aversion tested (γ = 1, 2, 4), except where cycles have no pull back to the trend. This is about floor-Kelly's own fractions. For historical Kelly the opposite held in simulation (half beat full), except in NO_REVERSION at γ = 1 with gold or cash.
8. **Stocks beat gold and cash as Bitcoin's partner**, but only at 2012 to 2026 returns. With stocks cut to 7% a year, cash led. The cash rate itself hardly matters.
9. **The floor zone was a 12-month buying signal in history.** From below 1.25× the walk-forward floor, every 12-month return was positive, and the median beat the trend's own growth by 1.42× to 1.46× (26 of 36 beat the trend over 12 months; over 48 months, half did). Month-ends between 1.25× and 1.6× the floor did as well (34 of 34 up, 1.77× the trend). This rests on three or four bear markets.
10. **A reflecting floor flatters Bitcoin retirement.** With the same shocks, reflecting the price at 0.432× trend, the floor Papers 15 to 17 used, raised Bitcoin only's SWR95 from 19.5% to 30.0% from today and from 5.0% to 6.5% from 2.5× trend (+10.5 and +1.5 points; about +54% and +30% of the rate). The withdrawal figures in Papers 15 to 17 rest on that assumption. A later study (F011) reran Paper 16's claims without it. Paper 16 sets yearly spending as a share of the floor's dollar rise over the next 12 months (its "coverage"; tiers at 80%, 100%, and 150% and 200% with its borrowing playbook). In Paper 16's own model, at its own start (the trend), a holder's safe spending at 99% confidence fell from 103.6% to 46.1% of the floor's growth.
11. **Against the pre-registered rules:**
    - P1 (saving) is mixed: history passes, the simulation fails.
    - P2 (retiring today) is not supported.
    - P3 (floor-Kelly ranked first) is mixed: first at full Kelly in history, last in simulation.
    - P4 (partner) is met by stocks, but the result depends on the sample.
    - P5 (buying signal) is supported.

# 1. Introduction

## 1.1 From a lump sum to a life cycle

Paper 19 tested a Kelly rule (Kelly, 1956; applied to investing by Rotando and Thorp, 1992) for Bitcoin, gold and cash on a lump sum: $1M at a start date, nothing added and nothing taken out. Its growth input for Bitcoin is **floor-anchored growth**, the log growth from today's price to the power-law floor four years ahead. Near the floor the input is high and the rule goes all in. Near and above the trend it

is low or negative and the rule moves into gold and cash. Walk-forward since 2014, full floor-Kelly ended with more Bitcoin than buy and hold in 98% of start and end pairs, but the edge over holding was not statistically established.

Real investors do not hold a lump sum. They add money while they work and take money out when they stop. Both change the question:

- **Accumulation.** Monthly contributions already average the purchase price (dollar-cost averaging, DCA; on its limits as an investment policy, see Constantinides, 1979). Does the rule's timing still add Bitcoin on top of that?
- **Distribution.** Withdrawals create sequence risk. Selling Bitcoin in a bear market to pay living costs locks in the loss. A rule that sells into bull runs and holds a gold, cash or stock sleeve could fund the bear years from that sleeve.

## 1.2 What the floor is used for

The Observatory defines the floor as a level price spends little time below and climbs back above quickly, not as a boundary that holds (`research/floor-definition.md`; Paper 20). The owner's reading is practical: the floor is a statistical lower bound, the level where most of the market starts to agree Bitcoin is extremely cheap. Price can close below it, but it does not stay there long, and that makes it a buying signal. A floor "works" over a period if price spends at most 5% of days below it and no visit lasts longer than 180 days.

Floor-Kelly uses the floor in that spirit. It does not forecast that price will be at the floor in four years. It asks what the growth would be if it were, and sizes the position on that cautious figure, in the spirit of robust Kelly rules that size on an unfavourable estimate rather than a central one (Rujeerapaiboon, Kuhn and Wiesemann, 2016; Sun and Boyd, 2018). Paper 19 placed the input near the 14th percentile of the model's own four-year forecast.

## 1.3 Questions

- **Q1. Saving.** With a fixed real contribution each month, does floor-Kelly end with more Bitcoin than putting every contribution into Bitcoin?
- **Q2. Retiring.** With a fixed real withdrawal each month, does floor-Kelly sustain a higher withdrawal rate, or survive bad starts better, than holding Bitcoin?
- **Q3. Which Kelly and which fraction.** Floor-Kelly, historical-average ("normal") Kelly, trend Kelly or trend-anchored Kelly; full, half or quarter?
- **Q4. Which partner asset.** Gold, cash only, or stocks?
- **Q5. The floor as a buying signal.**

# 2. Data and Method

## 2.1 Data

| Series | Source | Use |
|---|---|---|
| Bitcoin | `btc_historical.json`, daily closes 2010-07-18 to 2026-11-01 (analyses use closes to 2026-10-31); monthly series use the last daily close of each month | Prices; floor and trend fits use daily closes |
| Gold | GLD month-end close (`datasets/gold_monthly.csv`) | Investable gold after fees, as in Paper 19 |
| Stocks | SPY dividend-adjusted month-end close (`datasets/stocks_spy_monthly_yahoo.csv`) | Investable US stocks after fees. Monthly correlation 0.9998 with the S&P 500 total-return index; -0.10% a year |
| Cash | 3-month T-bill rate, FRED TB3MS | Cash return and the Kelly hurdle |
| Inflation | US CPI-U, FRED CPIAUCSL | Indexing contributions and withdrawals; real values |

Over 2012 to 2026, monthly log growth was 65% a year for Bitcoin, 13.8% for stocks and 5.7% for gold, with volatilities of 86%, 14% and 16%. Bitcoin's monthly correlation with stocks was 0.29 over the whole period and 0.53 since 2020; with gold it was 0.01 and 0.12. Inflation was 2.69% a year compounded and T-bills averaged 1.69%, a real cash return of about -1% a year.

## 2.2 The rule

At each month-end close *t*, using only data up to *t*, the rule picks long-only weights for Bitcoin and one partner asset; the remainder is cash. It maximises

$$w' m - w' \Sigma w / (2c), \text{ with } w \geq 0 \text{ and } w_{btc} + w_{partner} \leq 1,$$

where $c$ is the Kelly fraction (1 = full, 1/2 = half, 1/4 = quarter), $\Sigma$ is the covariance of the trailing 36 monthly log returns, and $m = g + \sigma^2/2 - \ln(1 + r_f)$. The partner's growth input is the expanding mean of its own monthly log returns since 2010-08. The code reproduces Paper 19's headline weights exactly (largest difference $2 \times 10^{-16}$) and its 1.26× lump-sum result.

### 2.3 Four Bitcoin growth inputs

| Code | Bitcoin growth input g(t) | Uses valuation? |
|---|---|---|
| FK, floor-Kelly | ln(Floor(t + 4 years) / P(t)) / 4, with the floor a τ = 0.01 quantile regression on daily closes up to $t$ | Yes, cautious |
| TA, trend-anchored Kelly | ln(Trend(t + 4 years) / P(t)) / 4, with the trend an OLS power-law fit on daily closes up to $t$ | Yes, central |
| HK, historical Kelly | The expanding mean of monthly Bitcoin log returns since 2010-08, × 12 | No |
| TK, trend Kelly | β(t) / age(t), the growth rate of the walk-forward OLS trend | No |

HK is the textbook plug-in. TK has the power law's growth but no view on where price stands. TA is the same rule as FK aimed at the central estimate instead of the floor, so FK against TA shows the cost or value of caution. It also captures how differently the two lines are estimated: the 1% quantile line moves after crashes far more than the OLS trend does.

### 2.4 Universes, benchmarks and costs

Each of the 12 rules runs with gold (G), with cash only (C) and with stocks (S): 36 rules in all. Benchmarks:

- **Bitcoin only.** Contributions buy Bitcoin; withdrawals sell it.
- **A matched mix for every rule:** a constant mix at the rule's average weights over the same window, rebalanced monthly. The rule's result against its own mix is its **timing value**. Because the mix is also rebalanced monthly, this nets out the growth that rebalancing alone can add (Fernholz and Shay, 1982; Bouchey et al., 2012).
- **In simulation:** fixed mixes (25%, 50% and 75% Bitcoin) and two non-Bitcoin baselines (100% stocks; 60% stocks and 40% cash).

Trading costs 0.25% of the value traded in Bitcoin, gold or stocks. Taxes are out of scope.

### 2.5 Cash flows

At each close the portfolio is marked, the cash flow applied and the portfolio rebalanced to the target weights.

- **Saving:** a fixed real contribution each month. Results are ratios, so the amount does not matter.
- **Retiring:** $1M at the start, then each month (rate × $1M) / 12, indexed to CPI (the convention of Bengen, 1994, and Cooley, Hubbard and Walz, 1998). A portfolio whose withdrawal exceeds its value is depleted.

Metrics:

- **Stack ratio:** final value against DCA into Bitcoin. At the same end price, this is the ratio of Bitcoin stacks.

- **Certainty-equivalent (CE) real wealth:** per unit contributed, for constant relative risk aversion $\gamma$ = 1, 2, 4. Full Kelly is growth-optimal for $\gamma$ = 1; a fraction near $1/\gamma$ is the classical match (MacLean, Thorp and Ziemba, 2010).
- **Investor experience:** the lowest value against money paid in, and maximum drawdown.
- **For retirees:**
  - survival;
  - SWR95 (the highest initial rate with at least 95% survival over 30 years);
  - the lowest real value reached.
- **Turnover:** the rebalancing trade alone, a year.

## 2.6 Historical tests

- **Saving:** every 48-month DCA window starting 2012-01 to 2022-10 (130), every 96-month window starting to 2018-10 (82), and every start to October 2026.
- **Retiring:**
  - every start at 4%, 8% and 12% to October 2026;
  - fixed 48-month windows at 8%;
  - the three complete cycle tops by monthly close (2013-11, 2017-12, 2021-10), each with the highest rate survived.
- **Buying signal:** forward returns from every month-end 2012-01 to 2025-10, by price against the floor known at the time.

Overlapping windows are not independent; there are about three independent four-year windows. History is described, not tested.

## 2.7 Simulated futures

The model is Paper 19's, extended to three assets and inflation:

- **Bitcoin:** the log residual to the published trend ($\log_{10}P = -16.493 + 5.688 \log_{10}$ days) follows an AR(1) fitted on monthly residuals 2012 to 2026 (coefficient 0.932 a month, half-life 9.8 months, long-run level 0.92× trend).
- **Shocks:** innovations are drawn in 12-month blocks jointly with gold, stock and CPI changes from the same months.
- **No reflecting floor.**
- **Cash:** 4.05%, the October 2026 T-bill rate.
- **Paths:** 2,000 of 30 years, seed 20261107, with the same paths for every rule.

Inside each path, the floor is refit every 12 months on monthly closes plus the path's own closes (primary). A secondary variant freezes it at the October 2026 walk-forward line. The trend for TA and TK is refit the same way.

| World | What changes |
|---|---|
| BASE | Cycles persist, as fitted |
| DAMPED | Shocks halved |
| FADING | Shocks shrink linearly to 10% by year 30 |
| NO_REVERSION | A random walk around the trend |
| BROKEN | Price grows at half the trend rate |

The start states are:

- **TODAY:** the actual October 2026 price, 0.49× trend and 1.16× the published floor.
- **TREND:** price at the trend.
- **PEAK:** price at 2.5× trend.

Five more cases are run from BASE:

- partner returns cut to 7% (stocks) and 4% (gold);
- cash at 2.65% and 1.69% (about 0% and −1% real), from TODAY;
- reflection at 0.432× trend, from TODAY and PEAK.

Intervals are 90% paired bootstrap intervals over paths (1,000 resamples). They measure simulation noise, not model uncertainty. Spreads of outcomes are given as 10th to 90th percentiles.

**The simulated world is harsher on the floor than history** (`analysis/f009_diagnostics.py`, 300 BASE paths; months below the floor and visit lengths from all 2,000 paths). The AR(1) was fitted as if monthly shocks were independent, but the shocks are drawn in 12-month blocks that keep their within-year runs. That probably explains part of why the simulated residual is more spread out than the historical one (standard deviation 0.346 against 0.283 in $\log_{10}$):

| Measure | History 2012 to 2026 | BASE simulation |
|---|---|---|
| 1st and 5th percentiles of price / trend | 0.40× and 0.44× | 0.21× and 0.29× |
| Share of month-end closes below the published floor | 2.2% | 16.3% |
| Longest visit below the published floor (history: daily closes; simulation: month-end closes) | 66 days (2015) | median 19 months, 90th percentile 37 months |
| Meets the Observatory floor standard (on daily closes: at most 5% of days below, no visit over 180 days) | Yes: 2.3% of days (123 of 5,418), longest visit 66 days | No; only DAMPED meets it (0.8% of months below, 90th-percentile longest visit 6 months). Simulated worlds are judged on month-end closes. |
| Deepest drawdown | -81% (month-end closes over 15 years) | median -94% (monthly, over 30 years) |

This affects every BASE result, Bitcoin only's withdrawal rate included, but it hurts rules anchored on a refit floor most. BASE is a harsher world than history, not a replay of it.

## 3. The Floor as a Buying Signal

**Table 1. Forward Bitcoin returns by price against the floor known at the month-end, 2012-01 to 2025-10**

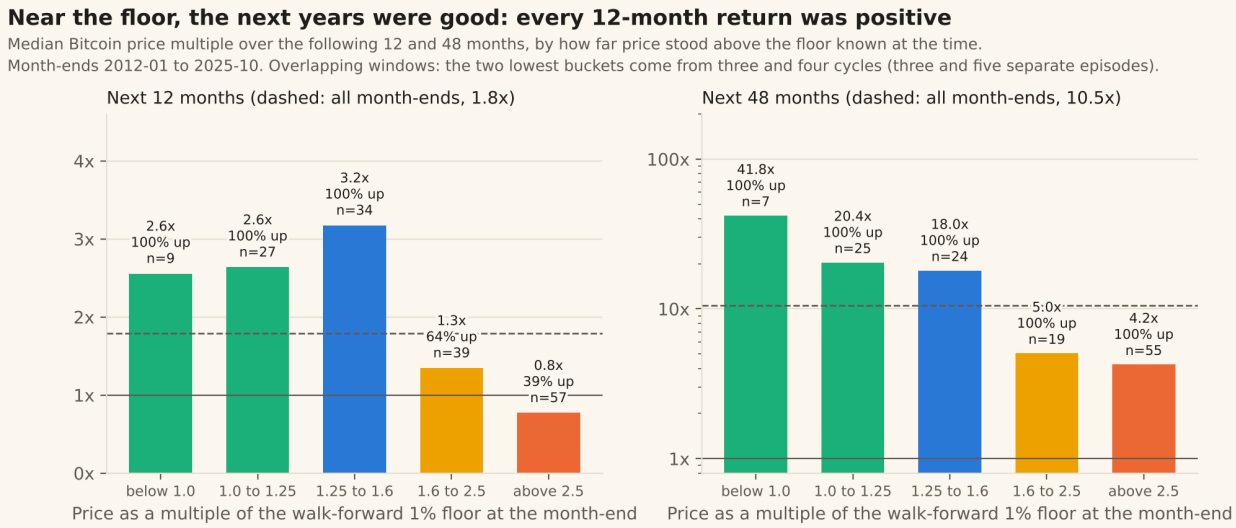
| Price / walk-forward 1% floor | Month-ends | Cycles (P5) / episodes | 12 months: median | 12 months: share up | 12 months: against trend growth | 48 months: median | 48 months: against trend growth |
|---|---|---|---|---|---|---|---|
| below 1.0 | 9 | 3 / 3 | 2.55× | 100% | 1.46× | 41.8× | 1.48× |
| 1.0 to 1.25 | 27 | 4 / 5 | 2.64× | 100% | 1.42× | 20.4× | 0.97× |
| 1.25 to 1.6 | 34 | 4 / 4 | 3.17× | 100% | 1.77× | 18.0× | 1.18× |
| 1.6 to 2.5 | 39 | 5 / 5 | 1.35× | 64% | 0.88× | 5.04× | 0.60× |
| above 2.5 | 57 | 4 / 4 | 0.77× | 39% | 0.49× | 4.25× | 0.43× |
| all month-ends | 166 | | 1.79× | 70% | 0.97× | 10.5× | 0.65× |

*Cycles: the cycle periods between monthly-close tops (2013-11, 2017-12, 2021-10, 2025-07) that the month-ends come from, as used for P5. Episodes: runs of month-ends in the bucket, merged when less than 12 months apart. "Against trend growth" divides the forward return by the walk-forward OLS trend's own growth over the same months. 48-month columns use month-ends to 2022-10: 7, 25, 24, 19, 55 and 130. Source:* `analysis/f009_results.json`*, H3.*

From the 36 month-ends below 1.25× the floor, **every following 12-month return was positive** (Table 1, Figure 1); the weakest was 1.29×. From all month-ends, 70% were. Raw multiples are inflated by Bitcoin's growth, so the fairer comparison is against the trend's own growth. On that measure, the floor zone beat the trend by a median 1.42× to 1.46× over 12 months, while all month-ends fell slightly behind it (0.97×). With the published floor the pattern is the same (1.37× to 1.83× over 12 months).

The trend × lowest-1% floor, the most stable floor in Paper 20, sits lower. Its bottom buckets show a weaker signal: 0.97× and 1.02× against trend over 12 months, from 17 month-ends. The signal belongs to the zone near the 1% line and the published floor, not to any line drawn lower.

The pre-registered test P5 is met. The median 48-month return from below 1.25× is 25× against 10.5× for all month-ends, and both low buckets draw on at least three bear markets (2011 to 2012, 2015, 2018 to 2019 and 2022). The sample is small and clustered: nine month-ends below the floor, from three episodes.




Source: Bitcoin Power Law Observatory, F009 (analysis/f009_figures.py). After 0.25% trading costs, before tax.

Figure 1. Forward Bitcoin returns by distance from the walk-forward floor.

# 4. Saving: Money In

## 4.1 History

**Table 2. DCA with Bitcoin, gold and cash: Bitcoin at the end against DCA into Bitcoin**

| Rule | Average Bitcoin | 48 months: median | 48 months: share ahead | 96 months: median | To Oct 2026: median | 48 months: against its own matched mix | Lowest value vs money paid in (worst window) | Median maximum drawdown |
|---|---|---|---|---|---|---|---|---|
| FK full | 84% | **1.10×** | **78%** | **1.27×** | **1.35×** | **1.23×** | 52% | -62% |
| FK half | 46% | 0.68× | 2% | 0.46× | 0.69× | 1.09× | 72% | -36% |
| FK quarter | 23% | 0.40× | 0% | 0.16× | 0.36× | 1.01× | 85% | -19% |
| TA full | 96% | 1.04× | 80% | 1.11× | 1.06× | 1.06× | 45% | -72% |
| TA half | 60% | 0.81× | 1% | 0.64× | 0.75× | 1.05× | 64% | -50% |
| TA quarter | 31% | 0.45× | 0% | 0.20× | 0.41× | 1.01× | 81% | -27% |
| HK full | 100% | 1.00× | 0% | 1.00× | 1.00× | 1.00× | 43% | -73% |
| HK half | 90% | 1.00× | 0% | 0.87× | 1.00× | 1.00× | 45% | -73% |
| HK quarter | 61% | 0.75× | 0% | 0.51× | 0.53× | 0.95× | 60% | -54% |
| TK full | 99% | 1.00× | 12% | 1.00× | 1.00× | 1.00× | 44% | -73% |
| TK half | 64% | 0.75× | 1% | 0.57× | 0.61× | 0.97× | 59% | -54% |
| TK quarter | 33% | 0.49× | 0% | 0.22× | 0.37× | 0.97× | 78% | -30% |
| Bitcoin only | 100% | 1.00× | | 1.00× | 1.00× | | 43% | -73% |

*FK floor-Kelly, TA trend-anchored Kelly, HK historical Kelly, TK trend Kelly; full, half and quarter are the Kelly fraction. 130 windows of 48 months (starts 2012-01 to 2022-10), 82 of 96 months, and 130 starts to October 2026. Source:* `analysis/f009_results.json`*, H1.*

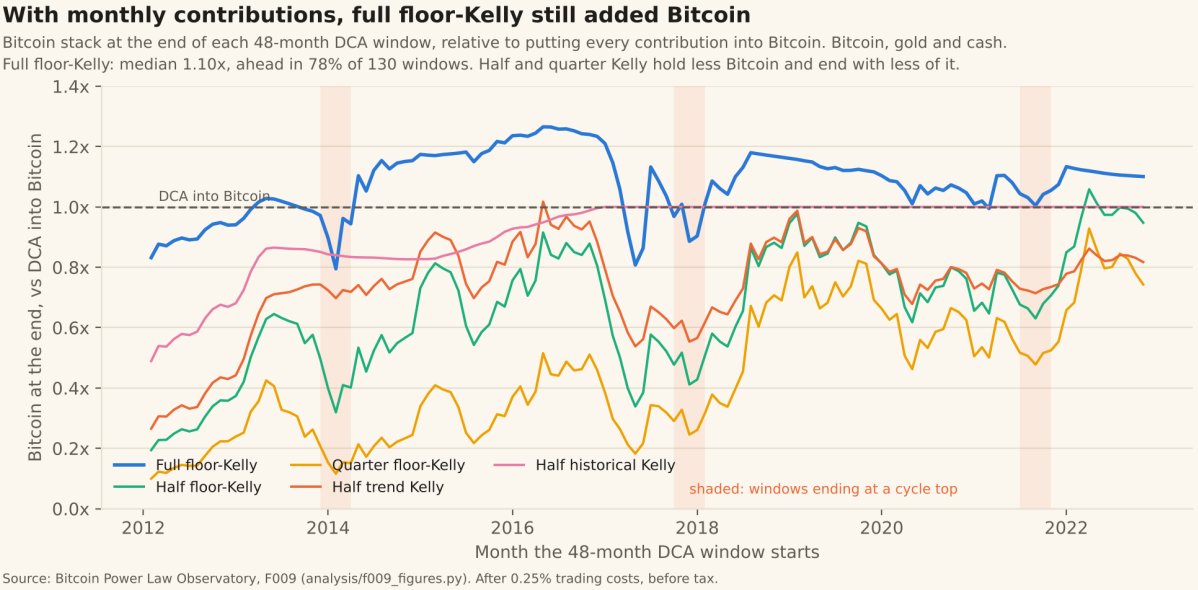
- **Full floor-Kelly adds Bitcoin on top of DCA, and the timing is the source.** It beat its own average mix by 1.23× over 48 months and 1.76× over 96 months, in 99% and 100% of windows. Figure 3 follows one saver, $1,000 a month from January 2014.
- **DCA shrinks the cost of ending at a top.** At past peaks, Paper 19's lump sum held only 0.49× to 0.70× of a holder's Bitcoin. With monthly contributions, windows ending within two months of a cycle top have a median of 0.97× and a worst of 0.79× (exploratory; Figure 2).
- **Normal Kelly is close to DCA into Bitcoin.** Bitcoin's average return since 2010 is so high that the plug-in formula held 100% Bitcoin at full Kelly. At half Kelly it averaged 90%. It held less than 100% Bitcoin in 59 months, all from December 2011 to October 2016, when trailing volatility was high, with a low of 58% in early 2012. Trend Kelly at full Kelly held 99%.
- **The valuation-aware rules have the most timing value.** Over 48 months, FK and TA beat their own mixes at every fraction (1.01× to 1.23×), while HK and TK did not (0.95× to 1.00×). Over 96 months, half and quarter trend Kelly and quarter historical Kelly also beat their mixes (1.10× to 1.19×), less than floor-Kelly (1.19× to 1.76×).
- **Lower fractions trade Bitcoin for a smoother ride.** Quarter floor-Kelly's worst point was 85% of money paid in, against 43% for DCA into Bitcoin, with a median drawdown of -19% against -73%. It ended with 0.40× the Bitcoin (Figure 4).

**Table 3. Certainty-equivalent wealth against DCA into Bitcoin, history (Bitcoin with gold unless marked)**

| Rule | 48 months: γ = 1 | γ = 2 | γ = 4 | 96 months: γ = 1 | γ = 2 | γ = 4 | Turnover a year |
|---|---|---|---|---|---|---|---|
| Floor-Kelly, full | 1.07× | 1.10× | 1.11× | 1.26× | 1.29× | 1.27× | 1.41 |
| Floor-Kelly, full, with stocks | 1.10× | 1.12× | 1.14× | 1.37× | 1.36× | 1.32× | |
| Floor-Kelly, half | 0.63× | 0.72× | 0.83× | 0.40× | 0.53× | 0.65× | 1.77 |
| Trend-anchored Kelly, full | 1.02× | 1.03× | 1.04× | 1.08× | 1.09× | 1.07× | 0.63 |
| Historical Kelly, half | 0.91× | 0.96× | 1.00× | 0.85× | 0.95× | 0.99× | 0.00 |
| Trend Kelly, half | 0.72× | 0.77× | 0.83× | 0.52× | 0.59× | 0.63× | 0.88 |

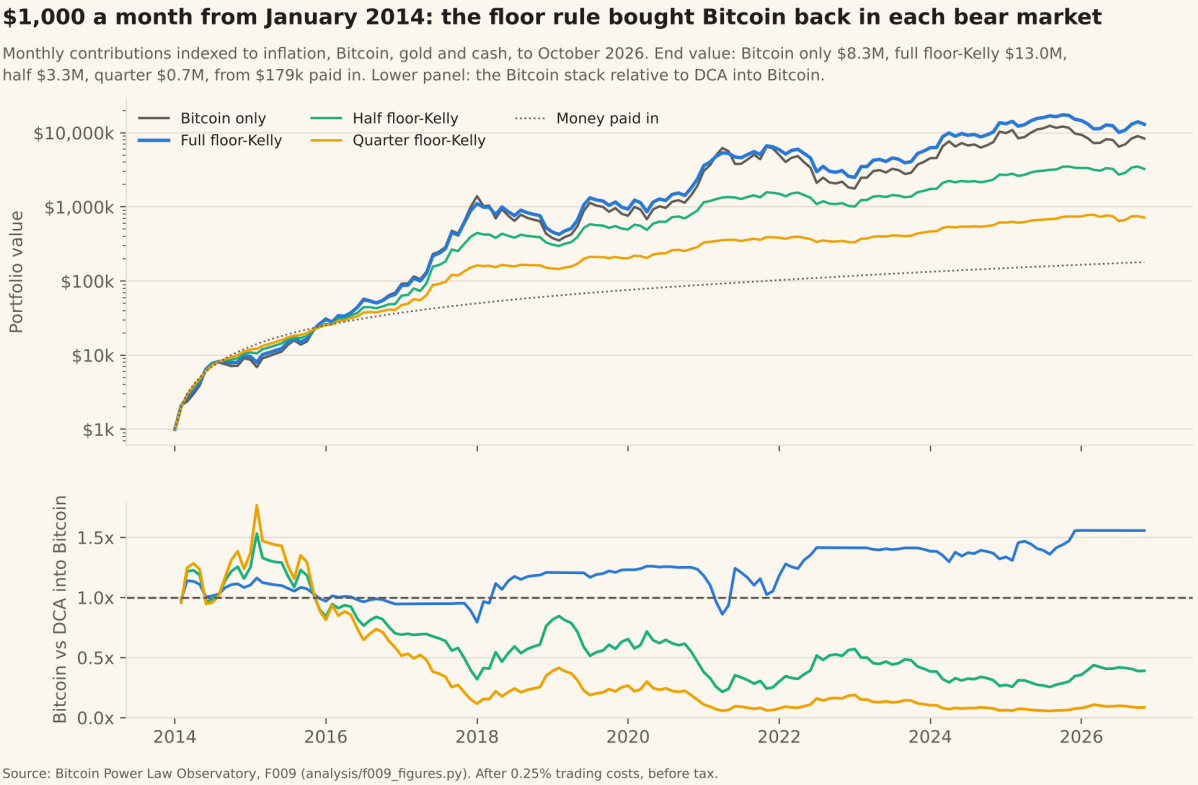
*Turnover is the median over 48-month windows. Source:* `analysis/f009_results.json`*, H1.*

In history, **full floor-Kelly with stocks had the highest certainty-equivalent wealth at every risk aversion tested** (γ = 1, 2 and 4; Table 3), followed by full floor-Kelly with cash or gold. Even a strongly risk-averse saver (γ = 4) did better with full Kelly than with half or quarter. Bitcoin rose so much over the period that holding less of it cost more than the smoother ride was worth.



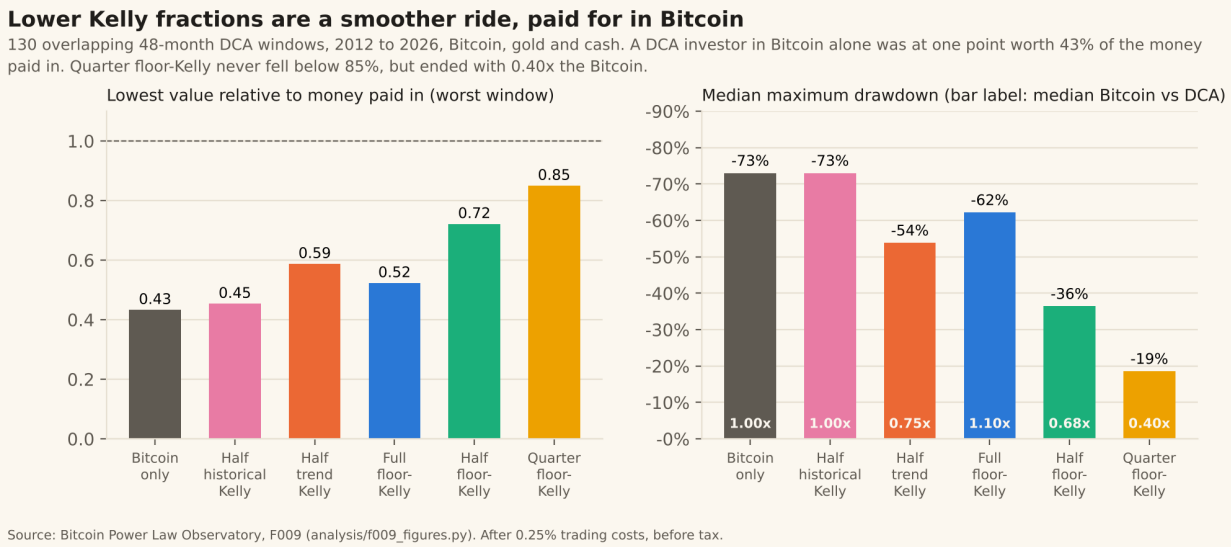

Source: Bitcoin Power Law Observatory, F009 (analysis/f009_figures.py). After 0.25% trading costs, before tax.

Figure 2. 48-month DCA windows: Bitcoin at the end against DCA into Bitcoin.




Source: Bitcoin Power Law Observatory, F009 (analysis/f009_figures.py). After 0.25% trading costs, before tax.

Figure 3. $1,000 a month from January 2014.




Source: Bitcoin Power Law Observatory, F009 (analysis/f009_figures.py). After 0.25% trading costs, before tax.

Figure 4. The ride: lowest value against money paid in, and drawdown.

## 4.2 Simulated futures

**Table 4. 20 years of DCA from today: Bitcoin at the end against DCA into Bitcoin (median, share of paths ahead), Bitcoin with gold**

| World | Floor-Kelly, full | Floor-Kelly, half | Floor-Kelly, full, frozen floor | Trend-anchored Kelly, full | Trend-anchored Kelly, half | Historical Kelly, half | Trend Kelly, full |
|---|---|---|---|---|---|---|---|
| BASE | 0.90× (47%) | 0.39× (16%) | 3.34× (91%) | 4.40× (96%) | 1.84× (77%) | 3.08× (96%) | 2.38× (95%) |
| DAMPED | 0.75× (33%) | 0.45× (8%) | 0.85× (39%) | 1.51× (85%) | 1.18× (66%) | 1.02× (61%) | 1.12× (76%) |
| FADING | 0.45× (20%) | 0.26× (3%) | 1.28× (65%) | 1.97× (89%) | 1.11× (58%) | 1.41× (85%) | 1.42× (92%) |
| NO_REVERSION | 0.15× (28%) | 0.11× (26%) | 1.00× (51%) | 1.00× (49%) | 0.57× (42%) | 1.08× (54%) | 1.08× (53%) |
| BROKEN | 3.01× (85%) | 1.65× (70%) | 4.34× (100%) | 4.85× (99%) | 5.52× (98%) | 3.79× (98%) | 3.01× (98%) |

*2,000 paths per world; the floor is refit inside each path except where marked frozen. Source:* `analysis/f009_mc_results.json`.

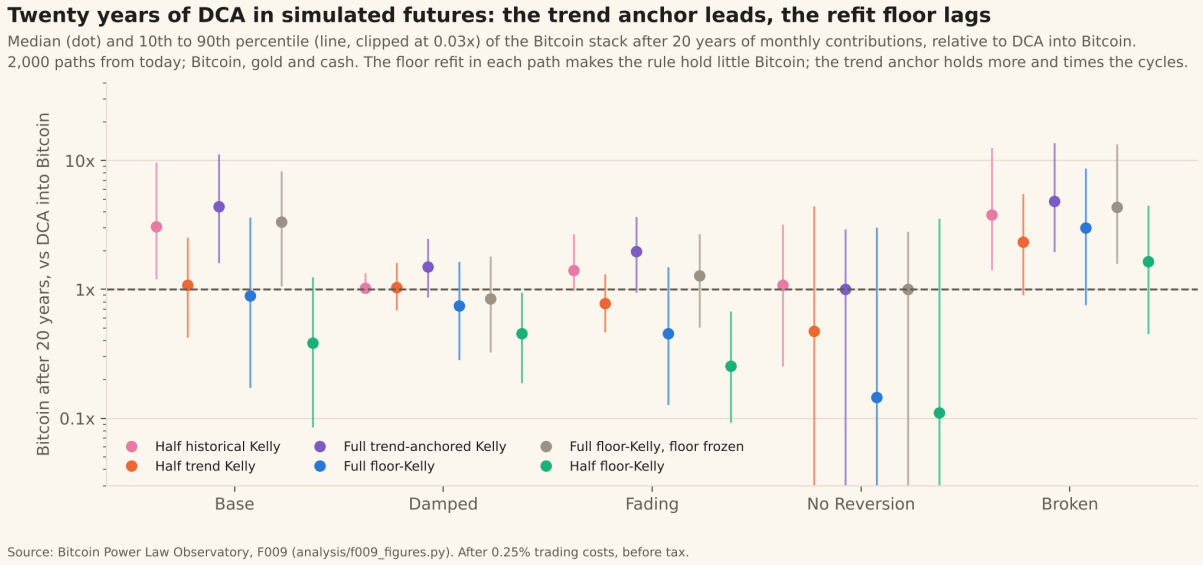
- **The registered test fails here** (Table 4, Figure 5). Full floor-Kelly ended at a median 0.90× in BASE (90% interval of the median 0.84× to 0.95×; 10th to 90th percentile of outcomes 0.18× to 3.57×) and below 1 in four of the five worlds. Only in BROKEN, where Bitcoin grows at half the trend rate, did it end ahead (3.01×).
- **Why: the refit floor falls (Figure 6).** In BASE, simulated crashes go deeper than history's (Section 2.7), and each one that sets a new low pulls the refit 1% line down. Measured as a multiple of the trend, the line falls from 0.45× today to 0.35× after five years, 0.27× after fifteen and 0.24× after twenty-five. Its slope barely moves (6.00 to 5.69). Both versions start all in. Both cut Bitcoin within about three years as price recovers from 0.49× to about 0.85× the trend: to 48% with the refit floor and 58% with the frozen one. After that, the refit version keeps falling

while the frozen one stays between about 50% and 58%: 44% against 58% after five years, 19% against 49% after twenty-five. The recovery toward the trend explains the early drop; the falling refit floor explains the gap. With the refit floor, price ends up further above the floor (a median 2.5× after five years and 3.3× after twenty-five, against 1.1× today). This is the mechanism Paper 19 (Section 3.9) found for floors refit inside a path.

- **The floor's slower growth with age adds to it (Figure 7).** Over the next four years the floor grows by β  ln((age + 4) / age) / 4 a year: about 30% a year today, 24% in 2031 and 12% in 2056. At any fixed distance from the floor the input falls with age. But the trend-based inputs shrink in the same way, so age alone does not explain the low weights. The level of the floor does.
- **The frozen floor does better because its level stays up.** Frozen at the October 2026 fit, the line grows at almost the same rate as the refit one (17.1% against 16.5% a year in 2041). Its level, though, rises from about 0.44× to 0.56× the trend after twenty-five years, instead of falling to 0.24×, so the rule holds more Bitcoin (3.34× in BASE). An investor cannot know in advance which line is right; refitting is what an investor would actually do.
- **Trend-anchored Kelly leads, much of it by construction.** It ended at 4.40× in BASE and above 1 in four of five worlds. Its starting target, the walk-forward trend four years out, is 0.898× the published trend. The model's own expected path for the same date is also 0.898×. In BASE, DAMPED and FADING it is therefore Kelly with the model's own forecast. It also led in BROKEN (4.85×), where the published trend is wrong, probably because its trend is refit inside each path and follows the slower growth. In NO_REVERSION, where there is no pull back to any trend, its edge disappears (1.00×).
- **In simulation, rules without valuation also beat DCA into Bitcoin.** Half historical Kelly ended at 3.08× in BASE, and its timing value against its own mix was 2.27× (Figure 13). This is consistent with Kelly's covariance term scaling the weight down after volatile spells, which in this model tend to precede crashes. We did not test that mechanism directly. In history the same rule had no timing value over 48 months (1.00×).

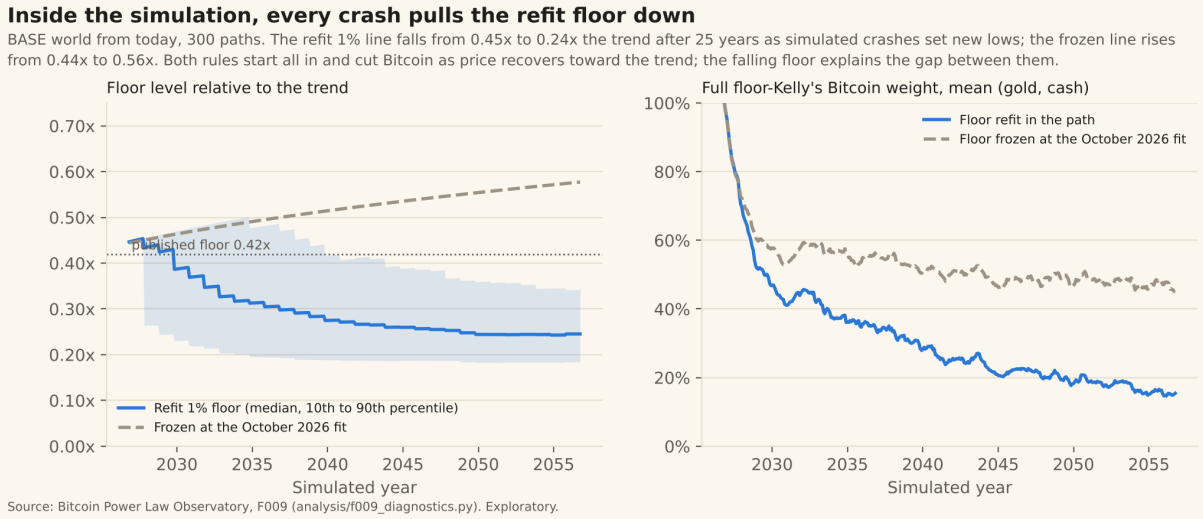
**Certainty-equivalent wealth from today.** In BASE, DAMPED and FADING the best candidate was full trend-anchored Kelly with stocks, a single winner at every risk aversion (CE 6.49×, 2.09× and 2.91× DCA into Bitcoin at γ = 1). Full floor-Kelly with gold was 0.83×, 0.72× and 0.45×. In BROKEN the

winner was half trend-anchored Kelly with stocks. In NO_REVERSION there was no single winner at γ = 1 (half historical Kelly and full trend Kelly with stocks); half trend Kelly with stocks won at γ = 2 and quarter trend Kelly with stocks at γ = 4.




Source: Bitcoin Power Law Observatory, F009 (analysis/f009_figures.py). After 0.25% trading costs, before tax.

Figure 5. Twenty years of DCA in simulated futures.




Source: Bitcoin Power Law Observatory, F009 (analysis/f009_diagnostics.py). Exploratory.

Figure 6. Inside the simulation, the refit floor falls relative to the trend (300 BASE paths).

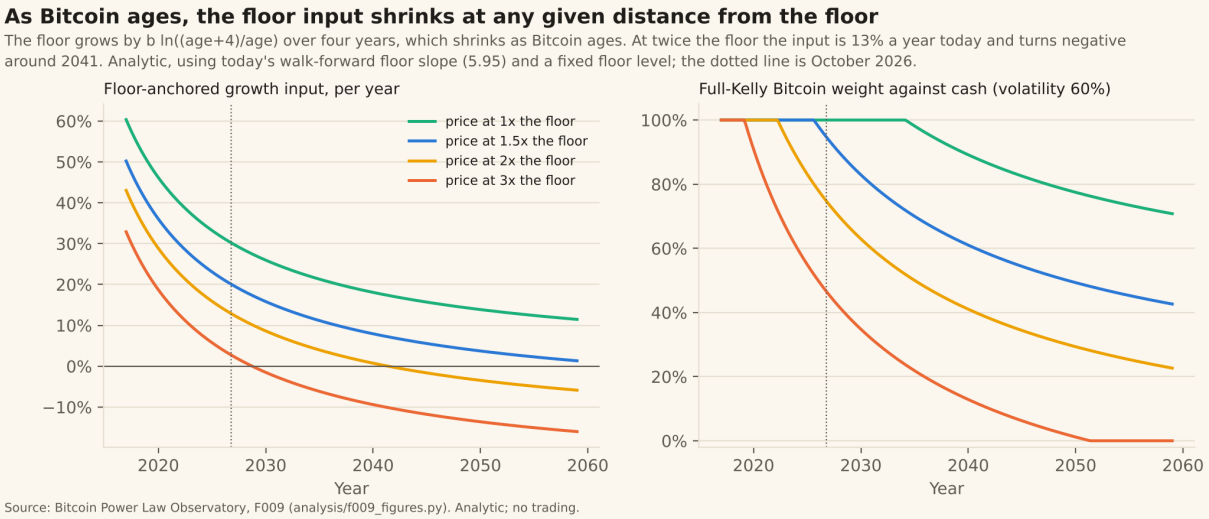



Figure 7. At a fixed distance from the floor, the floor input shrinks as Bitcoin ages.

# 5. Retiring: Money Out

## 5.1 History

From 2012 to 2026 almost any start survived almost any rate, because Bitcoin rose roughly 10,000-fold. At 4% and 8% every rule survived every start from 2012-01 to 2022-10. At 12%, Bitcoin only, historical Kelly and full trend Kelly were depleted from a few starts (98% to 99% survival); every floor-Kelly rule survived every start. The informative cases are retirements at a top.

**Table 5. Retiring at a cycle top, Bitcoin with gold and cash**

| Rule | Nov 2013: highest rate survived | Lowest real value at 8% | Dec 2017: highest rate | Lowest real value at 8% | Oct 2021: highest rate | Lowest real value at 8% | Oct 2021: real value in Oct 2026 at 8% |
|---|---|---|---|---|---|---|---|
| Bitcoin only | 9.7% | 11% | 12.3% | 19% | 12.6% | 19% | 0.38× |
| FK full | 16.1% | 25% | 19.3% | 32% | 17.3% | 28% | 0.85× |
| FK half | **19.1%** | **44%** | **20.0%** | **56%** | **20.8%** | **51%** | **1.00×** |
| FK quarter | 16.1% | 60% | 16.0% | 73% | 20.8% | 67% | 0.79× |
| TA half | 19.2% | 40% | 18.3% | 42% | 18.8% | 40% | 0.81× |
| HK half | 13.5% | 22% | 12.3% | 19% | 12.6% | 19% | 0.38× |
| TK half | 16.8% | 34% | 12.0% | 27% | 15.9% | 36% | 0.52× |

*FK floor-Kelly, TA trend-anchored Kelly, HK historical Kelly, TK trend Kelly. $1M invested at the close of the top month, withdrawals indexed to CPI, to the 2026-10 close (12.9, 8.8 and 5.0 years). "Highest rate survived" is on a 0.1% grid. These are survival rates over those windows, not 30-year rates. With stocks as the partner, half floor-Kelly survived 23.0%, 23.5% and 21.0%.*

**From every past top, half floor-Kelly survived about 19% to 21% a year, against 10% to 13% for Bitcoin only** (Table 5, Figure 9). At an 8% withdrawal from the October 2021 top, a Bitcoin-only retiree's real wealth fell to 19% of the start and stood at 38% five years later. Half floor-Kelly's fell to

51% and was back at 100%. At each top it held only 23% to 29% Bitcoin. At the 2021 top, for example, it held 25% Bitcoin and 75% gold: the 2022 withdrawals came from gold, and by November 2022, near the floor, it was back at 57% Bitcoin (Figures 8 and 10).

**Table 6. Fixed 48-month retirements at 8%, every start from 2012-01 to 2022-10**

| Rule | Lowest real value: median | Lowest real value: 10th percentile | Real value after 4 years: median | Lowest value above Bitcoin only's | End value above Bitcoin only's |
|---|---|---|---|---|---|
| Bitcoin only | 83% | 28% | 7.59× | | |
| Floor-Kelly, full | 86% | 37% | 9.43× | 60% | 88% |
| Floor-Kelly, half | 91% | 56% | 4.72× | 69% | 12% |
| Floor-Kelly, quarter | 91% | 69% | 2.04× | 78% | 6% |
| Trend-anchored Kelly, half | 89% | 48% | 5.87× | 69% | 15% |
| Trend Kelly, half | 90% | 44% | 6.11× | 70% | 10% |

*Source:* `analysis/f009_results.json`, *S1.*

Over fixed four-year retirements, half floor-Kelly roughly doubled the 10th-percentile low point (56% against 28%; Table 6). In the median window it ended with less, because Bitcoin rose so much over the period.

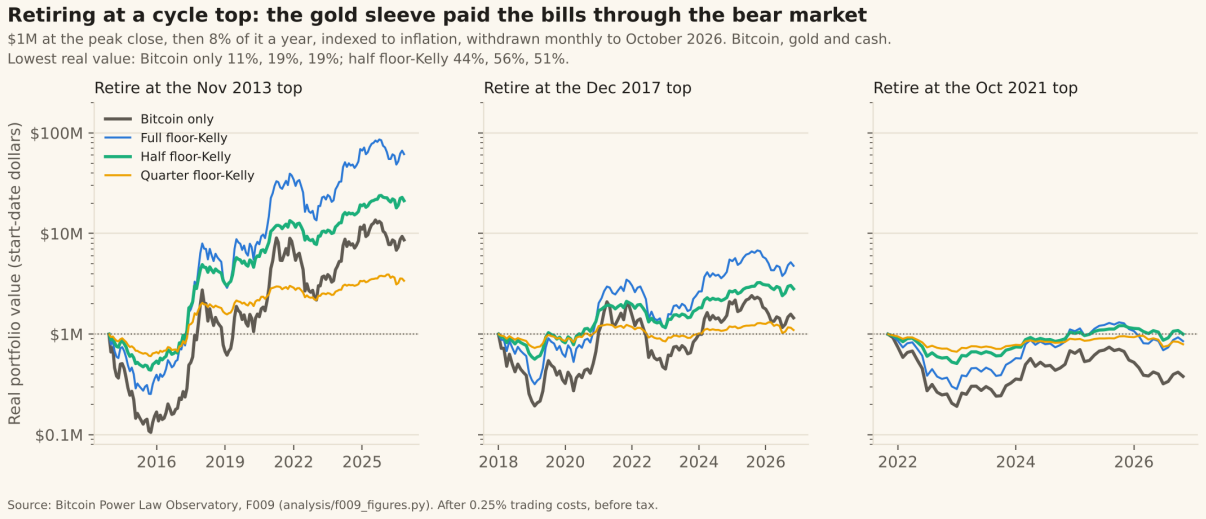



Figure 8. Retiring at a cycle top: real portfolio value at an 8% withdrawal.

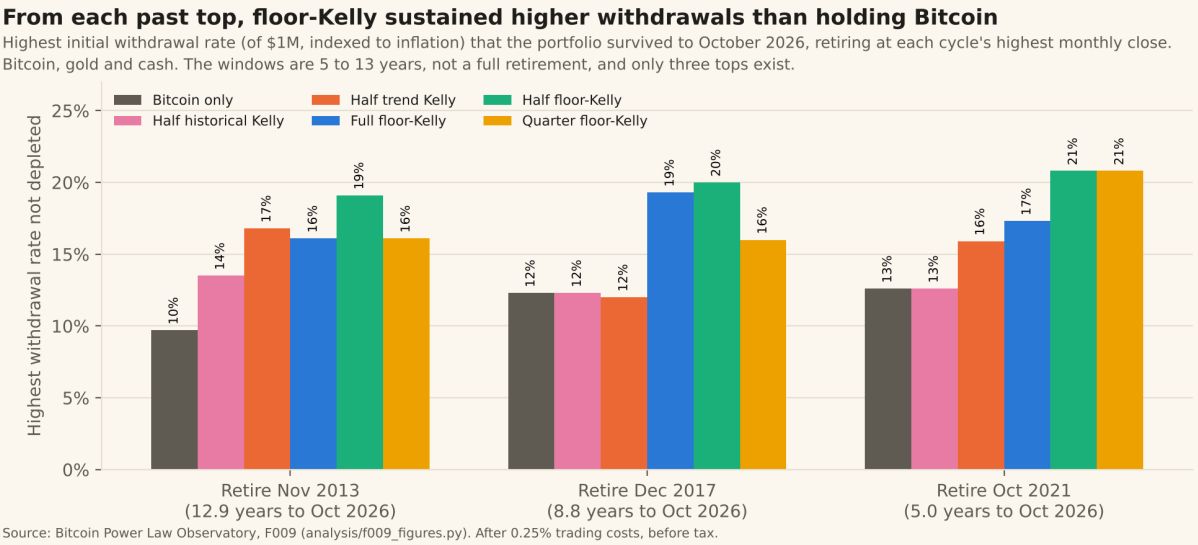



Figure 9. The highest withdrawal rate survived from each past top.

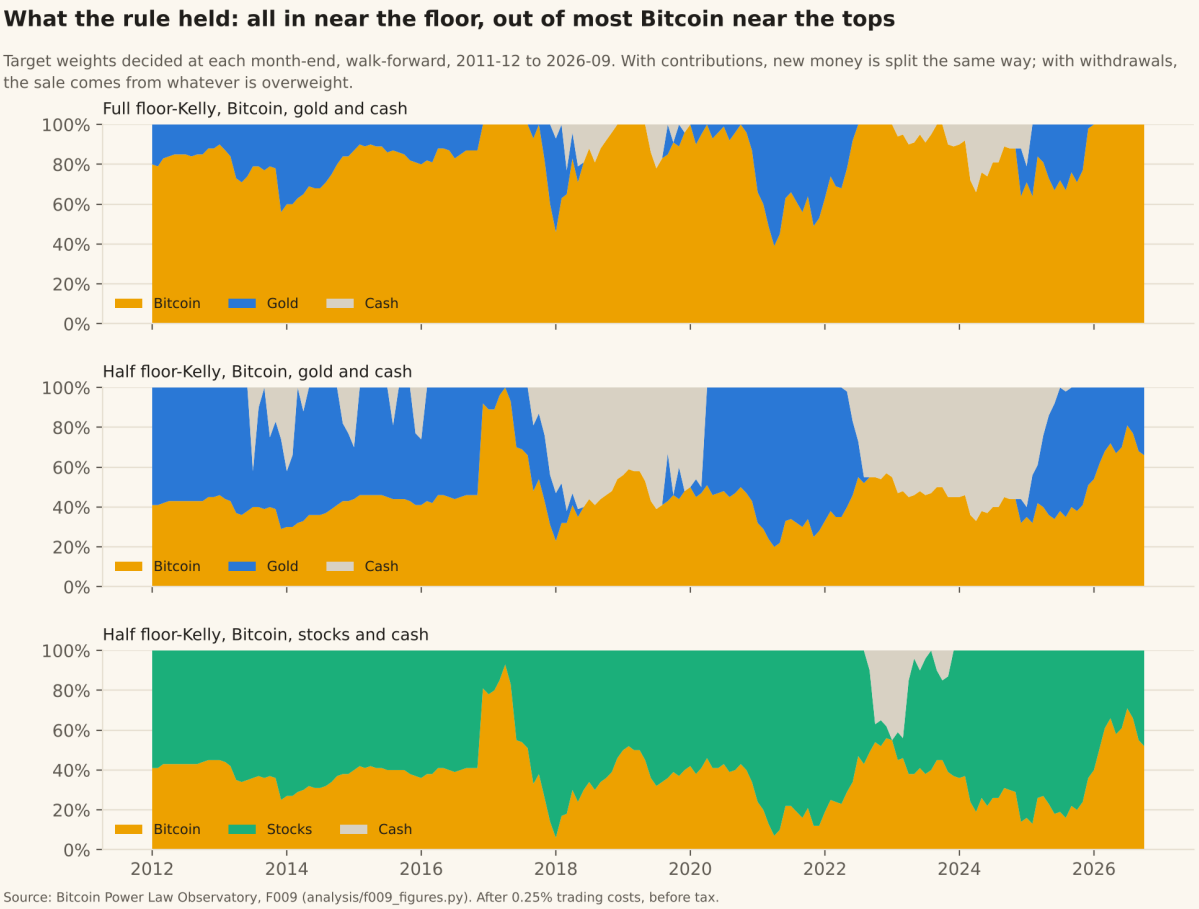



Figure 10. What the rule held, 2011 to 2026.

## 5.2 Simulated futures

**Table 7. Safe withdrawal rate (95% survival over 30 years), Bitcoin with gold and cash**

| World, start | Bitcoin only | Floor-Kelly, full | Floor-Kelly, half | Floor-Kelly, quarter | Trend-anchored Kelly, full | Trend-anchored Kelly, half | Historical Kelly, half | Trend Kelly, half |
|---|---|---|---|---|---|---|---|---|
| BASE, today | 19.5% | 14.5% | 10.5% | 7.5% | 21.5% | 20.0% | 21.0% | 17.5% |
| BASE, at trend | 11.0% | 10.0% | 8.5% | 6.5% | 12.0% | 13.0% | 11.5% | 11.0% |
| BASE, at 2.5× trend | 5.0% | 7.5% | 7.0% | 5.5% | 7.5% | 10.0% | 5.5% | 6.0% |
| DAMPED, today | 28.5% | 22.0% | 16.0% | 9.0% | 28.5% | 28.0% | 28.5% | 27.0% |
| FADING, today | 21.0% | 11.5% | 9.0% | 6.5% | 21.5% | 19.5% | 21.5% | 17.5% |
| FADING, at trend | 12.0% | 8.5% | 7.5% | 6.0% | 12.5% | 13.0% | 12.0% | 11.5% |
| FADING, at 2.5× trend | 5.5% | 6.5% | 6.0% | 5.0% | 7.5% | 10.0% | 5.5% | 6.0% |
| NO_REVERSION, today | 0.0% | 0.0% | 0.0% | 2.0% | 0.0% | 0.0% | 0.0% | 1.5% |
| BROKEN, today | 9.5% | 11.0% | 9.5% | 7.0% | 12.0% | 13.5% | 11.0% | 11.0% |
| BROKEN, at trend | 4.5% | 7.0% | 7.0% | 6.0% | 6.5% | 8.5% | 6.0% | 7.0% |
| BROKEN, at 2.5× trend | 2.0% | 5.0% | 5.5% | 5.0% | 3.5% | 6.5% | 2.5% | 4.0% |

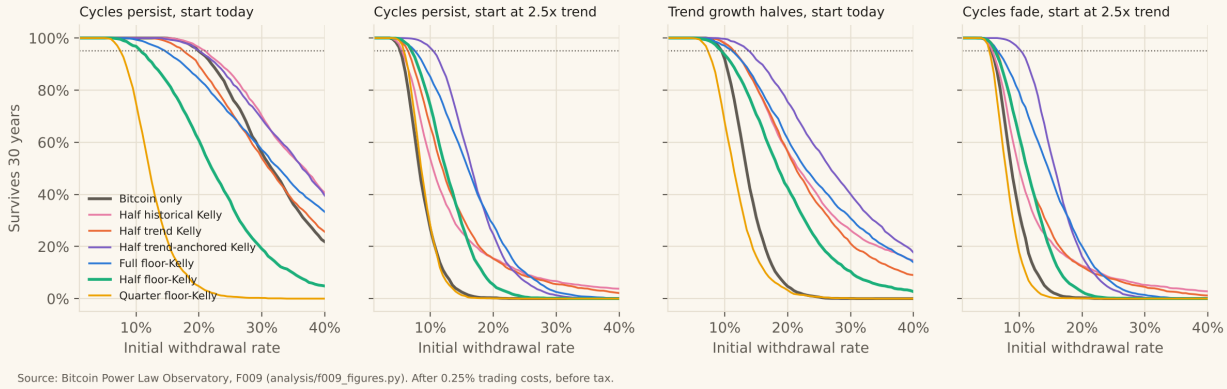
*0.5% grid from 1% to 40%. 0.0% means not even 1% survives in 95% of paths. Full tables for all three partners:* `analysis/f009_tables.md`.

- **From today, near the floor, Bitcoin only beats floor-Kelly.** In BASE, Bitcoin only's SWR95 was 19.5% and half floor-Kelly's 10.5%, a gap of -9.0 points (90% interval -9.5 to -9.0). Half floor-Kelly was at least equal in only two of five TODAY worlds (NO_REVERSION and BROKEN). Registered test P2 is not supported. Two things combine: Bitcoin only is fully invested in the cheapest asset, and the rule's shrinking floor input caps its Bitcoin at an average of 18%.
- **From a peak, half floor-Kelly beats Bitcoin only.** In BASE, FADING and BROKEN from 2.5× trend, half floor-Kelly's SWR95 was +2.0, +0.5 and +3.5 points higher (intervals +1.5 to +2.0, +0.5 to +0.5 and +3.5 to +4.0). That passes the 1-point bar in two of three worlds (S6). Its 10th-percentile low point in BASE was 54% of the start, against 15% for Bitcoin only.
- **The protection comes from holding less Bitcoin, not from the floor timing.** Half floor-Kelly's SWR95 from a peak equalled that of its own constant matched mix in BASE, was 0.5 point above it in FADING and 0.5 point below it in BROKEN. A fixed 50% Bitcoin, 50% gold mix did as well or better from every PEAK world (8.5%, 7.5% and 5.5%, against 7.0%, 6.0% and 5.5%), with a deeper 10th-percentile low point (42% against 54% in BASE). A later study (F011) found that half floor-Kelly on the published floor lowered safe spending from a peak in DAMPED, a world not run from a peak here.
- **The retirement rate depends on the start far more than on the rule.** Bitcoin only's SWR95 fell from 19.5% today to 11.0% at trend and 5.0% at 2.5× trend. No rule turns a peak retirement into a near-floor one (Figures 11 and 12).

- **In simulation, the trend-anchored rule usually ranks first for retirees.** Half trend-anchored Kelly with stocks was the recommended candidate from the BASE and FADING peaks. From the BROKEN peak, half floor-Kelly with stocks was recommended: it is not distinguishable from quarter trend-anchored Kelly with stocks (9.0% against 9.5%), and its 10th-percentile low point is 0.3 points higher. From a BASE peak, half trend-anchored Kelly with stocks had a 10th-percentile low point of 47%, against 71% for half floor-Kelly with stocks: it buys a higher rate with a deeper trough.
- **The simulated world is optimistic for everything.** Bitcoin's median 30-year multiple in BASE is 480×, because the power law is assumed to continue. 100% stocks had an SWR95 of 7.0% at the simulation pool's 13.7% return and 3.5% with returns cut to 7%. Bengen's historical figure is about 4%.

**Thirty-year survival by withdrawal rate in simulated futures**

2,000 simulated paths per world, Bitcoin, gold and cash, withdrawals indexed to simulated inflation. The floor is refit inside each path every year.
Dotted line: 95% survival. No reflecting floor: price can stay below any floor as long as the shocks take it.




Source: Bitcoin Power Law Observatory, F009 (analysis/f009_figures.py). After 0.25% trading costs, before tax.

Figure 11. Thirty-year survival by withdrawal rate in simulated futures.

**Safe withdrawal rate (95% survival over 30 years), by world and start**

Highest initial withdrawal rate, indexed to inflation, that survives 30 years in at least 95% of 2,000 paths. Bitcoin, gold and cash; floor refit in the path.
Bold: the best in each row. The grid runs to 40%.

| | Bitcoin only | Half historical Kelly | Half trend Kelly | Half trend-anchored Kelly | Full floor-Kelly | Half floor-Kelly | Quarter floor-Kelly |
|---|---|---|---|---|---|---|---|
| Base · today | 19.5% | **21.0%** | 17.5% | 20.0% | 14.5% | 10.5% | 7.5% |
| Base · at trend | 11.0% | 11.5% | 11.0% | **13.0%** | 10.0% | 8.5% | 6.5% |
| Base · at 2.5x trend | 5.0% | 5.5% | 6.0% | **10.0%** | 7.5% | 7.0% | 5.5% |
| Damped · today | **28.5%** | **28.5%** | 27.0% | 28.0% | 22.0% | 16.0% | 9.0% |
| Fading · today | 21.0% | **21.5%** | 17.5% | 19.5% | 11.5% | 9.0% | 6.5% |
| Fading · at trend | 12.0% | 12.0% | 11.5% | **13.0%** | 8.5% | 7.5% | 6.0% |
| Fading · at 2.5x trend | 5.5% | 5.5% | 6.0% | **10.0%** | 6.5% | 6.0% | 5.0% |
| No reversion · today | 0.0% | 0.0% | 1.5% | 0.0% | 0.0% | 0.0% | **2.0%** |
| Broken · today | 9.5% | 11.0% | 11.0% | **13.5%** | 11.0% | 9.5% | 7.0% |
| Broken · at trend | 4.5% | 6.0% | 7.0% | **8.5%** | 7.0% | 7.0% | 6.0% |
| Broken · at 2.5x trend | 2.0% | 2.5% | 4.0% | **6.5%** | 5.0% | 5.5% | 5.0% |

Source: Bitcoin Power Law Observatory, F009 (analysis/f009_figures.py). After 0.25% trading costs, before tax.

Figure 12. Safe withdrawal rate by world and start.

## 6. Which Kelly, and Which Fraction?

**Timing value (Figure 13).** Each rule is compared with its own matched mix.

- **History:** over 48 months only the valuation-aware rules gain. Floor-Kelly gains 1.23×, 1.09× and 1.01× at full, half and quarter Kelly; trend-anchored Kelly 1.06×, 1.05× and 1.01×. Historical and trend Kelly gain 0.95× to 1.00×. Over 96 months, half and quarter trend Kelly and quarter historical Kelly also gain (1.10× to 1.19×).
- **Simulation:** most rules gain in most worlds, including historical and trend Kelly, whose timing comes from the covariance term rather than from valuation. Quarter floor-Kelly gains in only two of five worlds, and half floor-Kelly falls short in BASE (0.93×).
- **NO_REVERSION:** the only world with no cycle to time. Every timing gain disappears or reverses there (0.50× to 1.04×).

**Applying the proposal's choice rules (Section 6.6 of the proposal).** The candidates are Bitcoin only and the 36 walk-forward rules. Identical candidates are merged; none were exactly identical on every path.

**Table 8. The highest certainty-equivalent wealth or SWR95 within each universe (before the proposal's tie-break)**

| Case | Best with gold | Best with cash | Best with stocks |
|---|---|---|---|
| History, 48 months, saver γ = 1 / 2 / 4 | Full FK (1.07× / 1.10× / 1.11×) | Full FK (1.08× / 1.08× / 1.07×) | Full FK (1.10× / 1.12× / 1.14×) |
| History, 96 months, saver γ = 1 / 2 / 4 | Full FK (1.26× / 1.29× / 1.27×) | Full FK (1.29× / 1.22× / 1.14×) | Full FK (1.37× / 1.36× / 1.32×) |
| BASE, DAMPED, FADING today, saver γ = 2 | Full TA | Full TA | Full TA |
| NO_REVERSION today, saver γ = 2 | Half TK | Half TK | Half TK |
| BROKEN today, saver γ = 2 | Half TA | Half TA | Half TA |
| BASE today, retiree | Full TA (21.5%) | Full TA (21.5%) | Full TA (21.5%) |
| BASE, FADING at 2.5× trend, retiree | Half TA (10.0%, 10.0%) | Half TA (9.5%, 9.5%) | Half TA (13.0%, 13.0%) |
| BROKEN at 2.5× trend, retiree | Quarter TA (7.5%) | Quarter TA (7.5%) | Quarter TA (9.5%) |

*Saver: certainty-equivalent wealth against DCA into Bitcoin. Retiree: SWR95. FK floor-Kelly, TA trend-anchored, TK trend Kelly. With the tie-break, the recommended retirees' candidates are half trend-anchored Kelly with stocks (BASE today) and half floor-Kelly with stocks (BROKEN at 2.5× trend). Full table:* `analysis/f009_tables.md`.

**For a saver.** Figure 14 shows certainty-equivalent wealth by Kelly fraction and risk aversion.

- **History (48 months):** full floor-Kelly with stocks is best at every γ, but there is no single winner. At γ = 1 and 2, full floor-Kelly with cash and with gold and full trend-anchored Kelly with stocks are within 5% of it; at γ = 4, full floor-Kelly with gold is.
- **History (96 months):** full floor-Kelly with stocks is the single winner at γ = 1 and 2, and is tied with full floor-Kelly with gold at γ = 4.

- **Simulation from today:** full trend-anchored Kelly with stocks is the single winner at every γ in BASE, DAMPED and FADING; half trend-anchored Kelly with stocks in BROKEN. In NO_REVERSION, no single winner at γ = 1 (half historical Kelly and full trend Kelly with stocks), half trend Kelly with stocks at γ = 2 and quarter trend Kelly with stocks at γ = 4.

**For a retiree.**

- **Today, simulation only:** the best SWR95 in BASE is 21.5%, reached by full trend-anchored Kelly with each partner. Not distinguishable from it are half historical Kelly and full trend Kelly with each partner, and half trend-anchored Kelly with stocks. The tie-break by the 10th-percentile low point picks half trend-anchored Kelly with stocks, by 0.6 points over the others, so in practice the tie-break hardly separates them.
- **At a top, history:** half floor-Kelly with stocks had the highest average rate survived (22.5%). This historical ranking is descriptive; the proposal registered no historical retiree rule.
- **At a top, simulation:** half trend-anchored Kelly with stocks is recommended in BASE and FADING, and half floor-Kelly with stocks in BROKEN.

**Do history and simulation agree?** The proposal's rule requires that a difference has the same sign and passes its threshold (5% for ratios, 1 point for SWR) in history and in at least three of the five TODAY worlds. For retirement at a top, we apply it to the three tops and at least two of the three PEAK worlds; this adaptation is ours.

**Table 9. Agreement between history and simulation**

| Comparison | History (48 months / 96 months, or the three tops) | Simulation (five TODAY worlds, or three PEAK worlds) | Verdict |
|---|---|---|---|
| Full against half floor-Kelly, gold, saver CE at γ = 2 | 1.52× / 2.44× | 1.99×, 1.55×, 1.56×, 0.86×, 1.58× | **Agree** |
| Full against quarter floor-Kelly, gold, saver CE at γ = 4 | 1.78× / 3.86× | 2.22×, 2.05×, 1.65×, 0.27×, 1.65× | **Agree** |
| Floor against trend anchor, full Kelly, gold, saver CE at γ = 2 | 1.06× / 1.18× | 0.21×, 0.47×, 0.21×, 1.02×, 0.71× | Disagree |
| Stocks against gold, full floor-Kelly, saver CE at γ = 2 | 1.02× / 1.06× | 1.48×, 1.25×, 1.74×, 1.70×, 1.49× | Disagree (history under 5% at 48 months) |
| Full floor-Kelly against half historical Kelly, gold, saver CE at γ = 2 | 1.14× / 1.36× | 0.30×, 0.61×, 0.27×, 0.56×, 0.81× | Disagree |
| Retiring at a top: half floor-Kelly against Bitcoin only, gold | +9.4, +7.7, +8.2 points | +2.0, +0.5, +3.5 points | **Agree** |
| Retiring at a top: half floor-Kelly against half trend-anchored, gold | -0.1, +1.7, +2.0 points | -3.0, -4.0, -1.0 points | Disagree |
| Retiring at a top: half floor-Kelly with stocks against gold | +3.9, +3.5, +0.2 points | +3.5, +4.0, +3.5 points | **Agree** (sample-dependent, Section 7) |

*Simulation order: BASE, DAMPED, FADING, NO_REVERSION, BROKEN; or BASE, FADING, BROKEN peaks. The history column for the tops shows the difference in the highest rate survived. For the tops the test is at least two of three tops and at least two of three PEAK worlds past 1 point, with the same sign.*

So the recommendations the plan allows are narrow:

- **savers using floor-Kelly:** full rather than half or quarter floor-Kelly;
- **retirees starting at a top:** half floor-Kelly rather than Bitcoin only (under the agreement test we adapted; in simulation a fixed 50/50 Bitcoin and gold mix did as well, Section 5.2), with stocks rather than gold at sample returns. For savers, stocks against gold did not pass the agreement test.

History and simulation disagree on whether to aim at the floor or the trend. The report makes no recommendation there.

**Normal Kelly.** In history, historical-average Kelly is close to Bitcoin only: at full Kelly it never sells, and at half Kelly it averaged 90% Bitcoin. Bitcoin's average return since 2010 is dominated by its early growth and falls only slowly. In simulation its half-Kelly version does better than Bitcoin only, through the covariance term rather than valuation. That gain depends on the model's mean reversion.

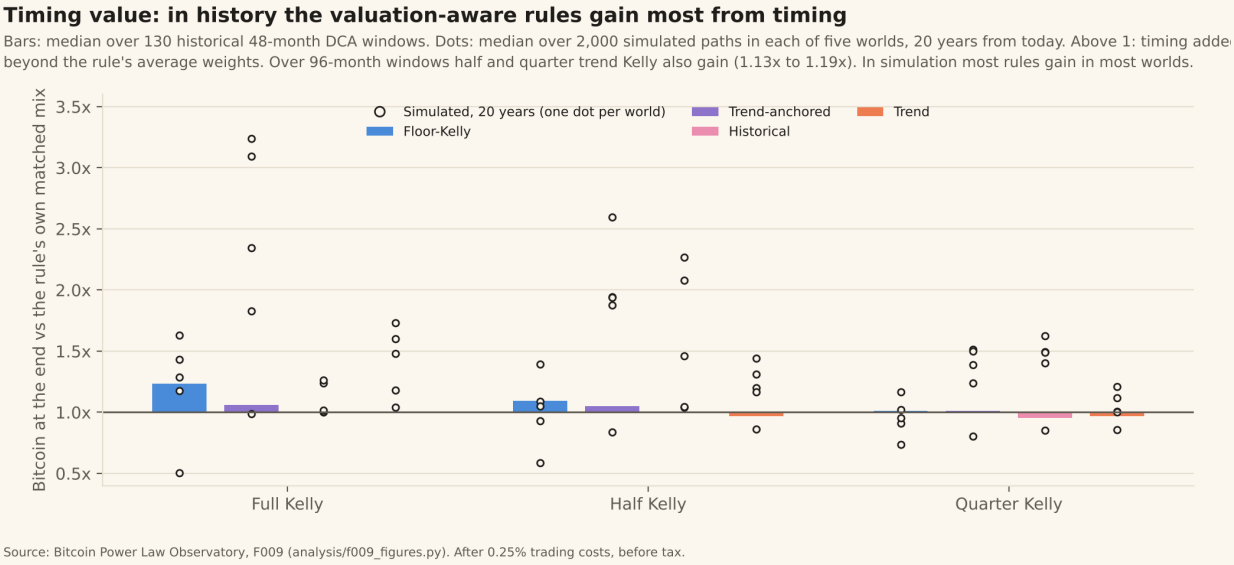



Figure 13. Timing value against each rule's own matched mix.

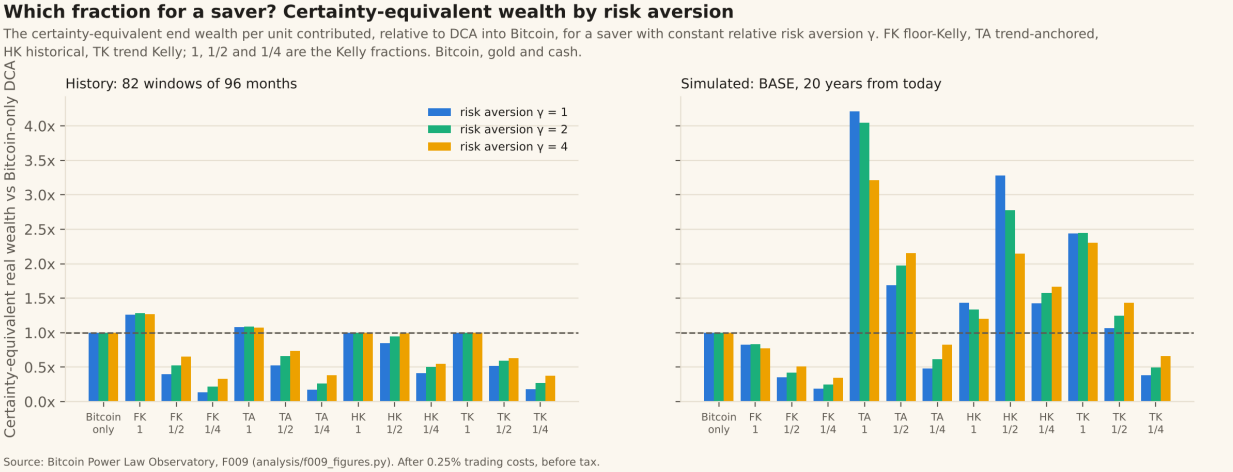



Figure 14. Certainty-equivalent wealth by fraction and risk aversion.

## 7. Which Partner Asset?

**Table 10. The floor-Kelly rules with each partner**

| Case | Partner | Full: 20-year DCA vs Bitcoin DCA | Half: SWR95 | Quarter: SWR95 | Half: 10th-percentile lowest real value at 4% |
|---|---|---|---|---|---|
| History, 48-month DCA (median) | gold / cash / stocks | 1.10× / 1.07× / 1.11× | | | |
| BASE, today | gold | 0.90× | 10.5% | 7.5% | 67% |
| BASE, today | cash | 0.99× | 10.5% | 7.5% | 64% |
| BASE, today | stocks | 1.09× | 13.5% | 10.0% | 69% |
| BASE, partner returns cut | gold | 0.82× | 10.0% | 7.0% | 67% |
| BASE, partner returns cut | cash | 0.99× | 10.5% | 7.5% | 64% |
| BASE, partner returns cut | stocks | 0.62× | 9.0% | 6.0% | 68% |
| FADING, today | gold / cash / stocks | 0.45× / 0.50× / 0.69× | 9.0% / 8.5% / 12.5% | 6.5% / 6.5% / 9.5% | |

*Partner returns cut: stocks shifted to 7% and gold to 4% a year in log terms. Source:* `analysis/f009_tables.md`.

- **Stocks first, but only at sample returns** (Table 10, Figure 15). Stocks ranked first on all three registered metrics: the historical median (1.11×), the BASE median (1.09×) and BASE SWR95 (13.5%). P4 is met by stocks. With stock returns cut to 7%, stocks fell to last on both simulation metrics (0.62× and 9.0%), and cash led. The preference is therefore **sample-dependent**, as the rule required us to say.
- **Gold against cash:** close in most cases (DAMPED is the exception: 19.5% with cash against 16.0% with gold). Gold earned more in the sample; cash has no drawdown.
- **The cash rate barely matters.** With cash at about 0% or -1% real instead of +1.4%, half floor-Kelly with cash lost 0.5 to 1.0 point of SWR95, and no ranking changed (S7).
- **Cash is sometimes first.** At the primary cash rate, half floor-Kelly with cash had the highest SWR95 of the three partners in DAMPED (19.5% against 18.5% and 16.0%), with partner returns cut, and with reflection.

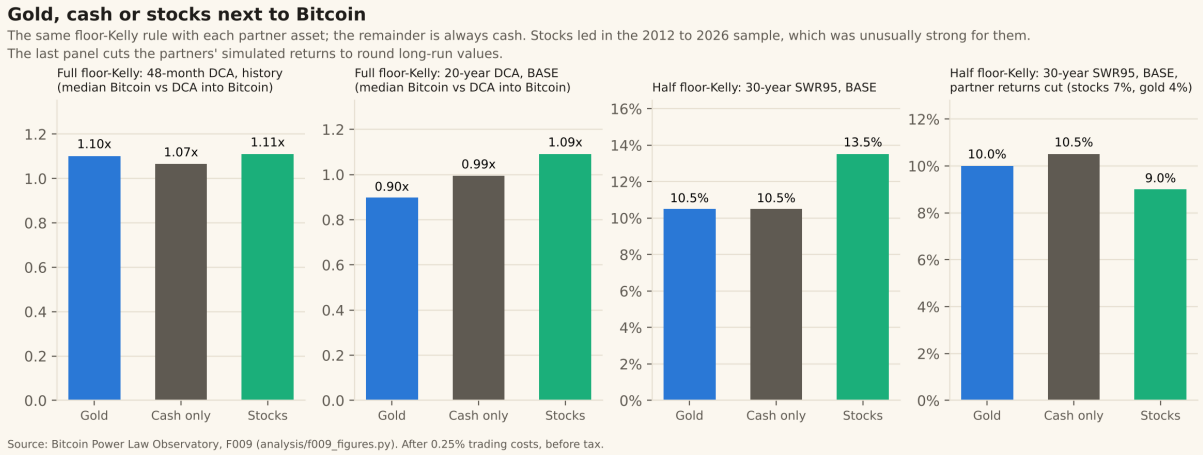



Figure 15. Gold, cash or stocks next to Bitcoin.

## 8. What a Reflecting Floor Assumes

Papers 15 to 17 estimated Bitcoin withdrawal rates in models where price reflects off a floor at 0.432× trend (the floor those papers used; the published floor is now 0.42×), as on the reflecting-barrier page (Paper 9). This study does not reflect. Analysis S3 runs the same shocks with and without reflection.

**Table 11. Reflection and the safe withdrawal rate (BASE, Bitcoin with gold)**

| Start | Rule | SWR95, no reflection | SWR95, reflecting at 0.432× | Change |
|---|---|---|---|---|
| Today | Bitcoin only | 19.5% | 30.0% | +10.5 points |
| Today | Half floor-Kelly | 10.5% | 21.5% | +11.0 points |
| 2.5× trend | Bitcoin only | 5.0% | 6.5% | +1.5 points |
| 2.5× trend | Half floor-Kelly | 7.0% | 9.0% | +2.0 points |

From today's price near the floor, reflection adds about 10 points to the 30-year safe withdrawal rate (Table 11, Figure 16). From a peak it adds 1.5 to 2.0 points, under the proposal's 3-point bar for "material", but rates there are small: 1.5 points is about 30% of Bitcoin only's 5.0%. From today, reflection also lifts Bitcoin only's 10th-percentile low point at a 4% withdrawal from 58% to 92% of the start. Without reflection, price can sit below the floor: 16% of simulated months in BASE, in visits whose median longest run is 19 months. With reflection, never.

Paper 15's 30-year SWR99 (the rate 99% of paths survive) of about $25k to $27k a year per Bitcoin under 3% inflation is about a third of today's price. Our reflecting case gives 28.0%, somewhat below that; without reflection it is 17.0%. By the proposal's 3-point threshold, set on SWR95, the effect from today (+10.5 points) is material. **The withdrawal rates in Papers 15 to 17 depend on the reflecting assumption.** This result opened registry item OQ17. A later study, F011 (forthcoming as Paper 23), reran Paper 16's claims without reflection. In Paper 16's own model, at its own start (the trend), a holder's safe spending at 99% fell from 103.6% to 46.1% of the floor's growth, and its 80%

"endowment" tier carried 12.3% ruin over 30 years instead of 0%. In F011's worlds, reflection also mattered from 2.5× trend (in BASE, holding's safe coverage was 70% without it and 100% with it). Paper 15's figures, and any in Paper 17, have not yet been rerun (registry item OQ19).

History sits between the two simulated worlds. Since 2012, price has spent about 2% of month-ends (2.3% of days) below the published floor, and its longest stay was 66 days. The non-reflecting BASE world has 16% of months below the floor, with a median longest stay of 19 months. The reflecting world has none. The owner's reading, that price "doesn't tend to stay there for long", describes history better than either model. This report does not change the earlier papers' claims; F011 does, for Paper 16. This section is the quantitative form of the owner's point that "reflecting barrier" is the wrong description of the floor.

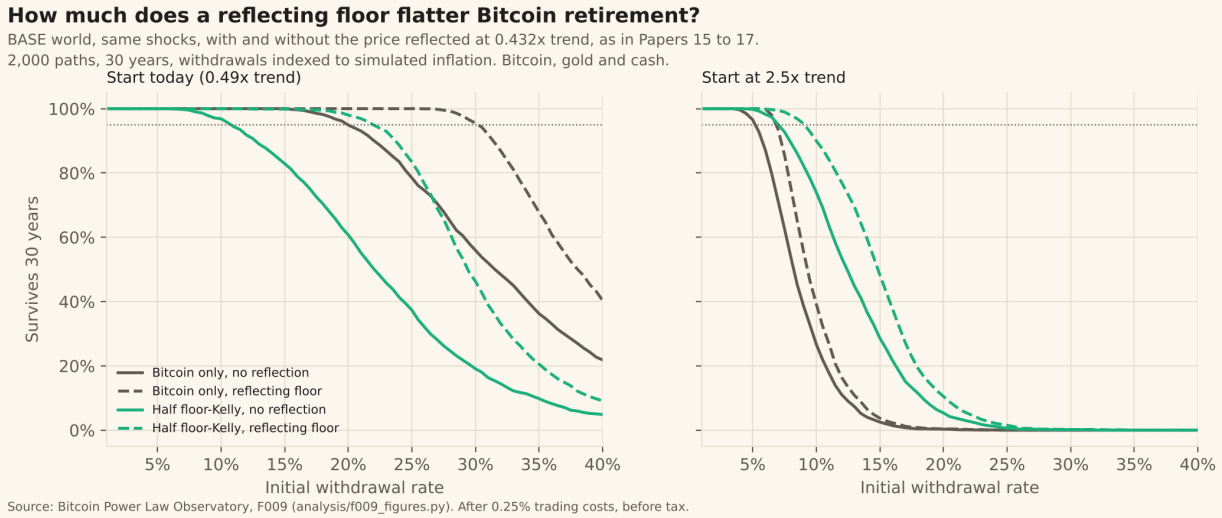



Figure 16. A reflecting floor and 30-year survival.

# 9. Scorecard Against the Plan

### Pre-registered tests (unchanged since commit 6ad98e5)

| | Test | History | Simulation | Verdict |
|---|---|---|---|---|
| P1 | Full floor-Kelly with gold against DCA into Bitcoin | Passes: 1.10×, 78% ahead, 99% ahead of its matched mix | Fails: 0.90× in BASE (0.84× to 0.95×) | **Mixed** |
| P2 | Half floor-Kelly with gold against Bitcoin only, retiring | Passes: higher end value at 8% in 3 of 3 tops | Fails: -9.0 points in BASE today; equal or better in 2 of 5 worlds | **Not supported** |
| P3 | Floor-Kelly first against historical and trend Kelly at each fraction | First at full Kelly only | Last of the three at full Kelly on BASE SWR95 (14.5% against 20.0% and 20.5%) | **Mixed** (first at full Kelly in history only; P3 registered no "not supported" condition) |
| P4 | Partner first on two of three metrics | Stocks | Stocks (lost when partner returns are cut) | **Stocks, sample-dependent** |
| P5 | Floor as a buying signal | 25× against 10.5× over 48 months | | **Supported** |

### Predictions made in the proposal before the simulation ran

- **Q1** (full floor-Kelly below 1 in BASE; below the historical 1.27× in every world): right for BASE, wrong for BROKEN (3.01×).
- **Q2** (Bitcoin only ahead from today in BASE; half floor-Kelly at least 1 point ahead from a peak in two of three worlds): right on both.
- **Q3** (FK and TA above 1.05× their mixes in at least three of five worlds, HK and TK within 0.95× to 1.05×): partly right.
  - **Right for FK at full Kelly and for TA at every fraction.**
  - **Wrong, narrowly, for half floor-Kelly:** above 1.05× only in DAMPED (1.40×) and FADING (1.09×); 1.048× in BROKEN. (On the first draw, v1.3, it passed with 1.051×; Section 12.)
  - **Wrong for quarter floor-Kelly** (above 1.05× only in DAMPED, 1.17×; with cash as the partner, in DAMPED and FADING).
  - **Right for HK at full Kelly** (within the band in three worlds), but **wrong for HK at half and quarter Kelly and for TK at every fraction.** Each fell outside the 0.95× to 1.05× band in at least three worlds, mostly above it.
- **Q4** (stocks' lead over gold shrinks by at least half when partner returns are cut; cash never first on SWR95 at the primary cash rate): half right.
  - **Right on the first part:** stocks' lead turned into a deficit.
  - **Wrong on the second:** cash was first in DAMPED, with partner returns cut, and with reflection.

# 10. Discussion

## 10.1 Why history and simulation disagree

The two tests ask different questions.

- **History asks** how the rule did while the power law was still being learned. The floor grew fast, crashes stayed above about 0.4× the trend, and the refit floor stepped down only in the 2015 and 2022 bear markets (Paper 20). The rule stayed mostly in Bitcoin and sold only near the tops.
- **The simulation asks** how the rule does if the published power law is right from here on, with shocks drawn from 2012 to 2026 in yearly blocks. In that world crashes go deeper than history's (Section 2.7). Each one pulls the refit floor down, so the rule measures growth from an ever lower floor and holds less and less Bitcoin (Figure 6). The trend-anchored rule plans on the model's own central path, which is the right plan inside the model, and much of its lead is built in.

So the choice between the floor and the trend as Kelly's target is a choice about **how much to trust the power law, and how deep the next crashes will be**:

- **If the trend is uncertain and crashes stay shallow**, as in 2012 to 2026, the floor's caution paid.
- **If the trend is right and crashes go deep**, as in the simulation, refitting the floor after every new low costs Bitcoin, and the trend anchor wins.
- **If the trend is wrong (BROKEN)**, both valuation-aware rules beat Bitcoin only, and the trend anchor still leads, probably because its refit trend follows the slower growth.
- **A later study (F010, forthcoming as Paper 22)** ran full floor-Kelly on the published floor, with no money in or out, in seven worlds built from history's own cycle shapes. After 10 to 14 years it held 1.42× to 1.48× a holder's Bitcoin where the price returned to the floor, 0.63× and 0.68× where the floor was missed, 1.01× in WILD, and 0.21× where multiples ratcheted up. Its edge needs the floor to keep being reached.

The frozen floor sits between these cases. Its level drifts up toward the trend and it holds more Bitcoin than the refit floor.

## 10.2 What this means for the floor

The owner's reading of the floor is supported where it was tested directly. Near the floor, every 12-month return in this sample was positive. After adjusting for the trend's own growth, the median still beat the trend over 12 months (1.42× to 1.46×), but not every time: 26 of 36 did, and over 48 months, 16 of 32. A level the market treats as "extremely cheap" is a good place to buy.

The rule that sizes positions on the floor four years out is a different claim. As a 12-month buy signal, the floor zone worked in this sample, and month-ends between 1.25× and 1.6× the floor did as well (34 of 34 up, 1.77× the trend). Over 48 months its edge over the trend's own growth was a coin flip (16 of 32). As the *expected* path for the next four years, it is too pessimistic once Bitcoin is older, unless the power law itself is in doubt.

### 10.3 A practical reading

The report makes no single recommendation where history and simulation disagree. Where they agree, the reading is:

- **Saving:** full floor-Kelly beat half and quarter floor-Kelly for savers at every risk aversion tested, in history and in four of five simulated worlds. Half and quarter buy a smoother ride (worst point 72% and 85% of money paid in, against 52% for full floor-Kelly and 43% for DCA into Bitcoin) at a large cost in Bitcoin. Whether to aim at the floor or the trend is the open question.
- **Retiring at a cycle top:** half floor-Kelly beat Bitcoin only in history and in two of three simulated peak worlds, and the two agree under the adapted test (Table 9). It did so mainly by holding less Bitcoin: in simulation, a fixed 50/50 Bitcoin and gold mix did as well or better (Section 5.2). From the three past tops, at an 8% withdrawal its real wealth fell at worst to 44% to 56% of the start, against 11% to 19% for Bitcoin only.
- **Retiring today, near the floor:** half floor-Kelly gave up about 9 points of safe withdrawal rate against Bitcoin only in BASE. Full trend-anchored Kelly did 2 points better than Bitcoin only. Half trend-anchored Kelly, half historical Kelly and full trend Kelly were 0.5 to 1.5 points above it. This case was not tested in history.
- **The partner:** stocks if one trusts 2012 to 2026 equity returns; otherwise gold or cash. Taxes, out of scope here, would make monthly rebalancing costly: the valuation-aware rules (floor-Kelly and trend-anchored Kelly) turn over 0.5 to 1.8 times the portfolio a year with gold in history, and up to 2.3 times with stocks.

### 10.4 Normal Kelly

The owner expected “normal” Kelly to over-allocate to Bitcoin, and it did.

- **At full Kelly:** on Bitcoin’s own history the plug-in formula never sold.
- **At half Kelly:** it averaged 90% Bitcoin, cutting its weight only in high-volatility months before 2017.

Bitcoin’s average return since 2010 is dominated by its early growth, and that average falls only slowly. Normal Kelly is close to buy and hold, as Paper 19 found for the lump sum.

## 11. Limitations

1. **Few independent episodes.** History has three complete cycles and about three independent four-year windows. The peak-start retirements are three cases of 5 to 13 years. History is described, not tested.
2. **The simulation is a model.** It assumes the published power law continues and that the cycles revert to it. Two consequences:
   - it rewards any rule that bets on that reversion (TA) or scales down after volatile spells (HK, TK);
   - its withdrawal rates are high for every rule (Bitcoin only 19.5% from today) because Bitcoin’s median 30-year multiple is 480×.

   The BROKEN and NO_REVERSION worlds show how much this matters.

3. **The floor is not a boundary (R026).** Price can close below any floor. The Observatory's caveat R026 puts the chance that price closes below the floor at least once within 30 years at 85% under a Poisson model. R026 predates the Observatory's floor definition, and its own source analysis is not in the repository, as Papers 19 and 20 also note. Since 2016 the published floor was visited 6 times in 10.8 years, none for longer than 7 days; on a Poisson model at that rate, at least one visit within 30 years is close to certain (Papers 19 and 20). The simulation lets price stay below as long as the shocks take it: 16% of simulated months sat below the published floor in BASE. The reflecting-floor ablation (Section 8) shows how much the opposite assumption is worth.
4. **Selection.** The floor input and horizon were chosen in Paper 19's search of 57 configurations. This study added rules but did not re-search the floor.
5. **Partner returns come from a strong sample.** The 2012 to 2026 stock return carries into every simulated path (the simulation pool's 13.7% a year). The partner-return sensitivity shows the stock preference does not survive lower returns.
6. **Costs and taxes.** Trading costs are 0.25%. Taxes are out of scope by owner decision. The valuation-aware rules (floor-Kelly and trend-anchored Kelly) turn over 0.5 to 1.8 times the portfolio a year with gold in history, and up to 2.3 times with stocks, so for a taxable investor monthly rebalancing would cost far more than modelled.
7. **Kelly ignores spending.** The rule is applied as is to a retiree with a fixed spending need. It is not the optimal consumption-investment policy (Merton, 1971).
8. **One metric did not work as designed.** "Bitcoin sold near the floor" counts all sales below 1.25× the floor, including rebalancing within the zone. Rebalancing rules sold more Bitcoin there than Bitcoin only in gross terms (for example 13.3 against 1.3 BTC per $1M in BASE at 8%). The metric does not show whether the sleeve shielded the stack, so it is reported but not used.
9. **Constant cash rate and one inflation path per shock draw.** Rates and inflation do not respond to the simulated economy.
10. **The simulated crashes are deeper than history's.** Shocks are drawn in yearly blocks, and the simulated residual is more spread out than the historical one (Section 2.7). BASE is a harsher world than history for every rule, and for refit-floor rules most. A later study (F011) scaled BASE's shocks to history's spread: 10.4% of months still sat below the floor, with a median longest visit of 12 months, so the wider spread explains only part of the gap. Floor-Kelly was not rerun in that world.
11. **Some mechanisms are inferred, not tested.** Two explanations are consistent with the results but were not tested directly:
    - the volatility-scaling explanation for historical and trend Kelly's simulated timing value;
    - the claim that trend-anchored Kelly leads in BROKEN because its refit trend adapts.

## 12. What Was Known When, and Deviations

- **2026-11-07, before any analysis:** questions, strategies, tests and decision rules P1 to P5 registered (commit 6ad98e5).
- **Historical tests H1 to H3, run next.** Their results were known before the research proposal.

- **A 60-path smoke test of BASE from today was seen.** Bitcoin only's SWR95 was at the 20% cap; half floor-Kelly's was about 10%; full floor-Kelly's DCA ratio was about 0.84×. The withdrawal grid was then extended from 20% to 40%, a logged deviation.
- **The owner asked for a research proposal before the research continued.** The first full simulation was stopped before any results were saved. Proposal v1.3 was accepted after independent review (8.0, 8.5, then 9.0/10).
  - It kept P1 to P4 unchanged and recorded P5 as decided.
  - It added TA, matched mixes for every rule, certainty-equivalent wealth, the 48-month retirement windows, the reflection and cash-rate runs, the adjusted signal and bootstrap intervals.
  - It registered predictions for the simulation (Section 9).
- **The simulation (16 cases × 2,000 paths) ran after the proposal was accepted.** No decision rule was changed after it ran.
- **v1.1, after an independent review of v1.0 (8.0/10):**
  - **Mechanism corrected.** The main mechanism behind the simulated result is now the refit floor falling relative to the trend in a world with deeper crashes (exploratory diagnostic `analysis/f009_diagnostics.py`, Figure 6), not the floor's growth rate.
  - **Comparison table added:** the simulated world against history (Section 2.7).
  - **Circularity stated in numbers, both ways.**
  - **Choice rules applied as written,** with the per-universe best and the agreement tests (Tables 8 and 9).
  - **Prediction scores corrected:** Q3 partly right, Q4 half right.
  - **Nine statements corrected.**
  - **Appendix added** with the remaining registered outputs.
- **v1.2, after the v1.1 review (8.5/10):**
  - the agreement claims are limited to what Table 9 tests (floor-Kelly's fractions; stocks against gold only for retirees at a top);
  - the mechanism is split into recovery toward the trend (early) and the falling refit floor (the gap between refit and frozen), with the frozen weights added to Figure 6;
  - small corrections in Sections 2.7, 8 and 9 and Limitation 11;
  - after the v1.2 review (9.0/10), three wording fixes: Limitation 10, the abstract's "Why", and "probably" for the BROKEN mechanism.
- **v1.3 (2026-11-10), for publication as Paper 21, after a fresh publication review of v1.2 (8.0/10, findings P21-1 to P21-21).** No analysis was rerun; every new number comes from the existing result files or from F010 and F011.
  - **Reflection (Section 8, Key finding 10, abstract):** the effect from a peak is now also stated as a share of the rate (about +27%); F011's rerun of Paper 16 without reflection is reported, and OQ17 is no longer described as open.
  - **Retiring at a top (Section 5.2, Key finding 3):** the protection is shown to come mainly from holding less Bitcoin; the matched-mix and fixed 50/50 mix results are reported, and the PEAK agreement test is labelled as adapted after the results.

- **The floor standard:** Section 2.7 tests history and the simulated worlds against the Observatory's floor standard (only DAMPED meets it); F011's BASE-CAL result is added to Limitation 10.
- **Corrected statements:** historical Kelly's weight cuts (59 months, December 2011 to October 2016); the floor signal after adjusting for trend growth (26 of 36 over 12 months, 16 of 32 over 48); P3's verdict (mixed); the NO_REVERSION saver winners; the turnover range; the stock return in the simulation pool (13.7%).
- **Added:** a Terms section, caveats in the abstract, F010's floor-Kelly results (Section 10.1), citations for every reference, and the floor-crossing wording of R026.
- **Layout:** figures and tables renumbered in order of appearance; lists and table captions fixed in the PDF.

• **v1.3, second round (2026-11-10), after a second publication review of v1.3 (9.2/10, findings P21b-1 to P21b-18).** No analysis was rerun.

- **Corrected:** the Q3 score for quarter floor-Kelly (above 1.05× only in DAMPED with gold); the retirees' worst real wealth in Section 10.3 (fell to 44% to 56% of the start, against 11% to 19%); the frozen floor's level after 25 years (0.56×, also in Figure 6's subtitle, redrawn from its saved data).
- **Sharpened:** the floor signal is a 12-month result (a coin flip over 48 months, and the 1.25× to 1.6× zone did as well); the floor standard in Section 2.7 is stated on daily closes (2.3% of days, longest visit 66 days); the 0.432× reflecting floor and Paper 16's coverage are explained; the abstract's retiree-at-a-top sentence carries its caveats.
- **Added:** in-text references to every figure and table, Table 1's 48-month counts, the data dates, F010's full results for floor-Kelly, the R026 context from Papers 19 and 20, and a page range for Bouchey et al.

• **v1.4 (2026-11-18): the simulation rerun after a correction to the block bootstrap.**

- **The defect.** The 12-month blocks of shocks could start only in the first 164 of the 176 pooled months, so the first and last 12 months were drawn less often (the very last month never). Both stretches had below-average shocks, so the drawn shocks averaged about 0.003 ($\log_{10}$) a month above the pool's mean. Through the AR(1), that set every simulated path about 10% higher against the trend than the fitted model says. It was found on 2026-11-17 while the engine was being ported to the site's Monte Carlo page, and fixed with circular blocks (a block may start at any month and wraps past the end). The fix is logged in `research/F009-preregistration.md`; `F009_BLOCKS=legacy` reproduces v1.3.
- **Rerun:** the simulation (16 cases × 2,000 paths, same seed) and the exploratory diagnostics. The historical tests do not use the draw and are unchanged.
- **What moved.** Simulated worlds are slightly harsher: BASE spends 16.3% of months below the published floor (was 13.0%), and Bitcoin's median 30-year multiple is 480× (was 517×). About three in four SWR95 values fall by 0.5 to 2 points, a quarter are unchanged and a few rise by up to 1 point (Bitcoin only from today 19.5%, was 21.0%; half floor-Kelly 10.5%, was 11.5%). Full floor-Kelly's 20-year DCA ratio in BASE is 0.90× (was 0.92×).
- **What did not.** P1 to P5 keep their verdicts, every agreement verdict in Table 9 is unchanged, and the reflection effect from today is still +10.5 points.
- **Changed readings:** the retiree tie-break in BASE today now picks half trend-anchored Kelly with stocks (was full trend Kelly with stocks); prediction Q3 is now also wrong for half floor-

Kelly (1.048× its matched mix in BROKEN, below the 1.05× bar); from a peak, the constant matched mix is within 0.5 point of half floor-Kelly in all three worlds; BASE today's best SWR95 (21.5%) is now shared by full trend-anchored Kelly with every partner (was 23.5%, with stocks alone); the BROKEN-peak tie set no longer includes half trend-anchored Kelly with stocks; reflection from a peak now adds 2.0 points for half floor-Kelly (was 1.5); Section 10.3's comparison of half trend-anchored, half historical and full trend Kelly with Bitcoin only now reads 0.5 to 1.5 points above it (was within 1 point); F011's figures cited in the abstract, Key finding 10 and Sections 5.2, 8 and 11 are its rerun values.

## Appendix: Remaining Registered Outputs

These outputs were registered in the pre-registration but are not discussed in the text. All come from `analysis/f009_tables.md` or the two result files.

**A1. DCA windows, Bitcoin with gold and cash: spread and endings near a top**

| Rule | 48 months: 10th percentile | 48 months: 90th percentile | Windows ending within 12 months after a top: median | Other windows: median |
|---|---|---|---|---|
| Floor-Kelly, full | 0.92× | 1.21× | 1.11× | 1.08× |
| Floor-Kelly, half | 0.37× | 0.92× | 0.66× | 0.68× |
| Floor-Kelly, quarter | 0.18× | 0.74× | 0.40× | 0.41× |

The registered split (ending within 12 months after a top) shows no shortfall at tops, because it mostly catches windows that end in the bear market after the top. The exploratory split in Section 4.1 (ending within two months of a top) shows the shortfall.

**A2. Retiring at the 2025-07 top (15 months to October 2026, incomplete)**

| Rule | Highest rate survived | Lowest real value at 8% | Real value at 8%, Oct 2026 |
|---|---|---|---|
| Bitcoin only | 53.4% | 45% | 0.54× |
| Floor-Kelly, full | 60.5% | 52% | 0.64× |
| Floor-Kelly, half | 79.4% | 76% | 0.88× |

**A3. Retirees' Bitcoin at October 2026 against Bitcoin only with the same withdrawals (median over starts 2012-01 to 2022-10)**

| Rule | 4% | 8% | 12% |
|---|---|---|---|
| Floor-Kelly, full | 1.46× | 1.48× | 1.52× |
| Floor-Kelly, half | 0.51× | 0.50× | 0.49× |
| Floor-Kelly, quarter | 0.14× | 0.12× | 0.10× |
| Trend-anchored Kelly, half | 0.66× | 0.64× | 0.61× |

**A4. Simulated SWR99 and censoring**

The SWR99 for every rule, world and start is in `analysis/f009_tables.md` ("MC SWR99"). In BASE today, Bitcoin only's SWR99 is 17.0% and half floor-Kelly's 8.0%; from 2.5× trend, 4.0% and 5.5%. Rules whose SWR95 would have been censored at the registered 20% cap are marked with an asterisk in the generated tables.

**A5. Simulated spread of 20-year DCA outcomes**

The 10th and 90th percentiles of the stack ratio for every rule and world, and the median real multiple of money paid in, are in `analysis/f009_mc_results.json`. For full floor-Kelly with gold in BASE they are 0.18× and 3.57×, and its median end value is 10.0× the real money paid in, against 11.0× for Bitcoin-only DCA.

## References


- Bengen, W. P. (1994). Determining withdrawal rates using historical data. *Journal of Financial Planning*, 7(4), 171 to 180.
- Bouchey, P., Nemtchinov, V., Paulsen, A. and Stein, D. M. (2012). Volatility harvesting: why does diversifying and rebalancing create portfolio growth? *Journal of Wealth Management*, 15(2), 26 to 35.
- Constantinides, G. M. (1979). A note on the suboptimality of dollar-cost averaging as an investment policy. *Journal of Financial and Quantitative Analysis*, 14(2), 443 to 450.
- Cooley, P. L., Hubbard, C. M. and Walz, D. T. (1998). Retirement savings: choosing a withdrawal rate that is sustainable. *AAII Journal*, 20(2), 16 to 21.
- Fernholz, R. and Shay, B. (1982). Stochastic portfolio theory and stock market equilibrium. *Journal of Finance*, 37(2), 615 to 624.
- Kelly, J. L. (1956). A new interpretation of information rate. *Bell System Technical Journal*, 35(4), 917 to 926.
- MacLean, L. C., Thorp, E. O. and Ziemba, W. T. (2010). Long-term capital growth: the good and bad properties of the Kelly and fractional Kelly capital growth criteria. *Quantitative Finance*, 10(7), 681 to 687.
- Merton, R. C. (1971). Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory*, 3(4), 373 to 413.
- Rotando, L. M. and Thorp, E. O. (1992). The Kelly criterion and the stock market. *American Mathematical Monthly*, 99(10), 922 to 931.
- Rujeerapaiboon, N., Kuhn, D. and Wiesemann, W. (2016). Robust growth-optimal portfolios. *Management Science*, 62(7), 2090 to 2109.
- Sun, Q. and Boyd, S. (2018). Distributional robust Kelly gambling. arXiv:1812.10371.
- Bitcoin Power Law Observatory:
  - Paper 9 (The Reflecting Barrier);
  - Paper 15 (The Bitcoin Safe Withdrawal Rate at Any Entry Residual);
  - Paper 16 (Passive vs Active Retirement);
  - Paper 17 (Playbook Optimization);
  - Paper 19 (Kelly Sizing With Floor-Anchored Growth, v1.3);
  - Paper 20 (Does a Frozen Power-Law Floor Hold?, v1.2);

- F010, Sell Into Euphoria, Buy Back at the Trend (`report/ladder_rebuy.md`; forthcoming as Paper 22);
- F011, Spend the Floor's Growth, Bank the Euphoria (`report/spend_floor_growth.md`; forthcoming as Paper 23);
- Research Claims Registry v3.0 (Domain 8, C8.1 to C8.8, records this report's claims).

## Data and Reproducibility

- **Data:**
  - `btc_historical.json`, daily closes 2010-07-18 to 2026-11-01 (analyses use closes to 2026-10-31);
  - monthly GLD, SPY and ^SP500TR (Yahoo via Firecrawl, retrieved 2026-11-06 and 2026-11-07);
  - FRED TB3MS and CPIAUCSL.
- **Power law:** logA = -16.493, β = 5.688, genesis 2009-01-03 (C1.3). Published floor 0.42× trend.
- **History:** `python3 analysis/f009_cashflow.py` writes `analysis/f009_results.json` and the window CSVs (about 45 seconds).
- **Simulation:** `python3 analysis/f009_mc.py 2000 4` writes `analysis/f009_mc_results.json` (about 45 minutes on 4 cores, seed 20261107).
- **Tables:** `python3 analysis/f009_tables.py > analysis/f009_tables.md` generates every table in this report from the two result files.
- **Figures:** `python3 -W ignore analysis/f009_figures.py` writes `figures/f009/`, each figure with a CSV of its data.
- **Plan:** `research/F009-preregistration.md` and `research/F009-proposal.md`.